Final Exam
Math 3 – Daugherty
March 13, 2012

Name (Print): _____

Last   First

On this, the Final Math 3 exams in Winter 2012, I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature: _____

Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided. Take a moment now to print your name and section clearly on your Scantron form and on the cover of your exam booklet and sign the affirmation. You may write on the exam, but you will only receive credit on Scantron (multiple-choice) problems for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 25 multiple choice problems worth 1 each, for a total of 25.

1. If $f(x) = |x - 4|$, what can you say about

$$\lim_{h \to 0} \frac{f(4 + h) - f(4)}{h}?$$

(a) It doesn't exist because the denominator is going to 0, so $\dfrac{f(4 + h) - f(4)}{h}$ goes to infinity.

(b) It doesn't exist because $\lim\limits_{h \to 0^+} \dfrac{f(4 + h) - f(4)}{h} \neq \lim\limits_{h \to 0^-} \dfrac{f(4 + h) - f(4)}{h}$.

(c) It's equal to ± 1.

(d) It's equal to 0.

(e) None of the above.

2. Which of the following is true if

$$f(x) = \begin{cases} x^3 + x + 1 & x > 0 \\ \cos(x) & x \leq 0 \end{cases}?$$

(a) This function is continuous and differentiable everywhere.

(b) This function is continuous everywhere, and differentiable everywhere except at $x = 0$.

(c) This function is continuous everywhere except at $x = 0$, but is differentiable everywhere.

(d) This function is continuous and differentiable everywhere except at $x = 0$, where it is neither.

(e) None of the above.

0+0+1 = 1   3(0)^2 + 1 = 1
cos(0)=1   -sin(0) = 0
continuous!   not differentiable!

3. Let $f(x) = \dfrac{x^2 - 4}{x^2 - 2x}$. Which of the following is true?

(x+2)(x-2)

x(x-2)

(a) This function has one horizontal asymptote and one vertical asymptote.

(b) This function has one horizontal asymptote and two vertical asymptotes.

(c) This function has no horizontal asymptotes and one vertical asymptote.

(d) This function has no horizontal asymptotes and two vertical asymptotes.

(e) None of the above.

H: x=0, V: y=1

4. Use the linear approximation of $\ln(x)$ at $a = 1$ to estimate $\ln(5)$.

(a) 2

(b) 4

(c) 5

(d) $1 + \frac{1}{5}(4)$

(e) None of the above.

y' = 1/x

L(x) = 1/1(x-1) + ln(1) = x-1
L(5) = 4

5. Which of the following describes the graph of $f(x) = x^3 + 6x^2 + 9x$?

  (a) It is positive exactly over $(0, \infty)$,
      increasing exactly over $(-\infty, -3)$, and
      concave up exactly over $(-2, \infty)$.

  (b) It is positive exactly over $(0, \infty)$,
      increasing exactly over $(-\infty, -3) \cup (-1, \infty)$, and
      concave up exactly over $(-2, \infty)$.

  (c) It is positive exactly over $(-\infty, -3) \cup (0, \infty)$,
      increasing exactly over $(-\infty, -3) \cup (-1, \infty)$, and
      concave up exactly over $(-2, \infty)$.

  (d) It is positive exactly over $(-\infty, -3) \cup (0, \infty)$,
      increasing exactly over $(-\infty, -3) \cup (0, \infty)$, and
      concave up exactly over $(-2, \infty)$.

  (e) None of the above.

      f = x(x+3)^2 >0 when x>0
      f' = 3x^2 + 12 x + 9 = 3(x+3)(x+1) >0 when x>-1 and x<-3
      f" = 6x+12 = 6(x + 2) > 0 when x>-2

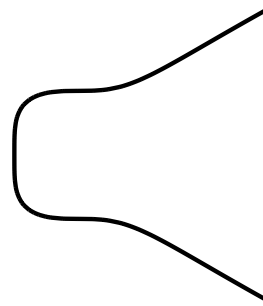6. First ask yourself how many points on the curve

$$y^3 + x^4 = -1$$

have vertical or horizontal tangent lines, respectively. Now, based on that information, which of the following could be a graph of this curve (plotted on the usual x-y axes, but shown without those axes)?
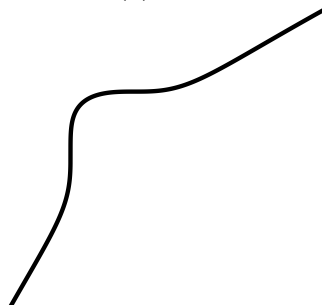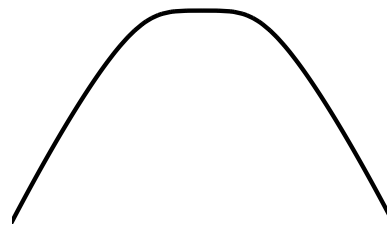
(a)
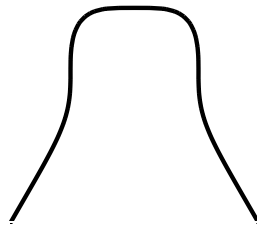
(b)

(c)

(d)

(e)

None of the above.

7. What is the maximal value of $(1 + x^2)^{1/2}$ on $[-1, 2]$?

   (a) 0

   (b) 1

   (c) $\dfrac{2}{\sqrt{5}}$

   (d) $\sqrt{5}$

   (e) None of the above.

   f' = x/sqrt(1 + x^2), so the only c.p. is at x=0

   f(0) = 1
   f(-1) = sqrt(2)
   f(2) = sqrt(5) MAX!

8. Suppose a parked car is leaking oil in such a way that the oil is leaving a very thin circular puddle. If the area of the puddle is growing at a rate of 3 cm^2/hr, how fast is the puddle's radius growing when the puddle is 20cm wide $(r = 10cm)$?

   (a) $\frac{1}{20\pi}$ cm/hr

   (b) $\frac{3}{20\pi}$ cm/hr

   (c) $\frac{3\pi}{5}$ cm/hr

   (d) 60π cm/hr

   (e) None of the above.

   A' = 3
   want r'(10)
   A = pi*r^2
   so A' = 2pi*r * r'
   so when r=10, 3= 2pi*10*r', and so r' = 3/20pi

9. Suppose a ball is tossed upwards at a velocity of 7m/s from a height of 2 meters above the ground. Neglecting wind/air resistance, what is happening to the ball exactly 1 second later? (The acceleration due to gravity has a magnitude of 9.8 m/s)

(a) It is still moving upwards.

(b) It is falling, but is still above its initial position.

(c) It is falling, but is below its initial position.

(d) It has already hit the ground, so we don't know for sure.

(e) None of the above.

$y(0)=2, y'(0)=7$
$y' = -9.8t + 7, y=-4.9t^2 + 7t + 2$
$y'(1) <0, y(1) >2$

10. If $f(x) = 5^x - \ln(3x)$, what is $f'(x)$?

   (a) $f'(x) = 5^x - \frac{1}{\ln(3x)}$

   (b) $f'(x) = 5^x - \frac{1}{x}$

   (c) $f'(x) = \ln(5)5^x - \frac{1}{x}$

   (d) $f'(x) = \frac{5^x}{\ln(5)} - \frac{1}{3x}$

   (e) None of the above.

11. If $f(x) = \cos(x)e^{\sec(x)+2}$, what is $f'(x)$? (be mindful of small simplifications)

   (a) $f'(x) = -\sin(x)e^{\sec(x)\tan(x)}$

   (b) $f'(x) = (\cos(x) - \sin(x))e^{\sec(x)+2}$

   (c) $f'(x) = \left[\cos(x) * (\sec(x)\tan(x) + 2) - \sin(x)\right]e^{\sec(x)+2}$

   (d) $f'(x) = (\tan(x) - \sin(x))e^{\sec(x)+2}$

   (e) None of the above.

(-sin(x) + cos(x)sec(x)tan(x))exp(sec(x)+2)

12. Which of the following is equal to $\int_0^4 \arctan(x)\ dx$?

(a) $\sum_{i=1}^{4} \arctan(i/4)$ no limit

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \boxed{\frac{1}{n}} \arctan(i/n)$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \arctan(4i/n)$

(d) $\lim_{n\to\infty} \boxed{\sum_{i=0}^{n}} \frac{4}{n} \arctan(4i/n)$

(e) None of the above.

13. Approximate $I = \int_2^8 x\cos(x)\ dx$ using three equally spaced rectangles and left-sided endpoints.

(a) $I \approx 2\cos(2) + 4\cos(4) + 6\cos(6)$

(b) $I \approx 4\cos(4) + 6\cos(6) + 8\cos(8)$

(c) $I \approx 4\cos(2) + 8\cos(4) + 12\cos(6)$

(d) $I \approx 4\cos(2) + 8\cos(4) + 12\cos(6) + 16\cos(8)$

(e) None of the above.

Dx = 2, evaluate at 2,4,6

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14. Assume a population of lemmings is growing at a rate proportional to the number present. If there are 50 lemmings now and 110 lemmings in 6 months from now, now many lemmings to you expect there to be in a year?

(a) $50\left(\dfrac{11}{5}\right)$

(b) $50\left(\dfrac{11}{5}\right)^2$

y = Ae^(kt)
y(0) = 50
y(6) = 110

(c) $50\left(e^{2*\frac{11}{5}}\right)$

(d) $110\left(\dfrac{11}{5}\right)$

(e) None of the above.

15. Solve $\dfrac{dy}{dx} = y^3$

-1/(2y^2) = x+ c
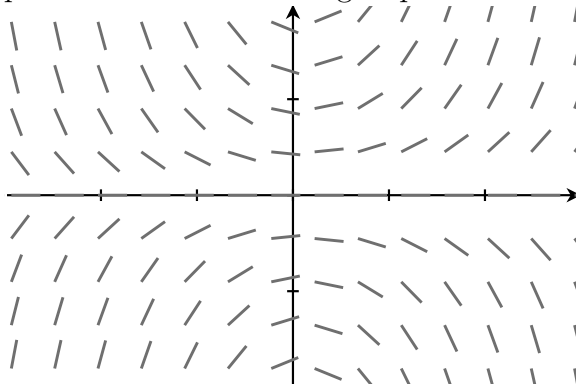
(a) $x = \dfrac{1}{4}y^4 + C.$

(b) $y = \dfrac{1}{\sqrt{-2(x+C)}}$

(c) $y = \dfrac{1}{\sqrt{-2x}} + C$

(d) $y = \sqrt[3]{e^{x+C}}$

(e) None of the above.

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16. Which of these differential equations has the following slope field?

(a) $\dfrac{dy}{dx} = x + y$

(b) $\dfrac{dy}{dx} = x - y$

(c) $\dfrac{dy}{dx} = xy$

(d) $\dfrac{dy}{dx} = \dfrac{x}{y}$

(e) None of the above.

17. Use Euler's method with step size $\Delta x = 2$ to approximate $y(4)$ if

$$\frac{dy}{dx} = y - x \qquad \text{and } y(0) = 5.$$

(a) $y(4) \approx 2$

(b) $y(4) \approx 41$

(c) $y(4) \approx 43$

(d) $y(4) \approx 115$

(e) None of the above.

x0 = 0, y0=5, m0 = 5, y1= 5+ 2*5 = 15
x1 = 2, y1 = 15, m1= 15-2=13, y2= 15 + 2*13 = 41.

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18. If $f'(x) = \sin(x) + x^{-1} + e^{5x}$, which of the following could be $f(x)$?

    (a) $f(x) = \cos(x) - x^{-2} + 5e^{5x}$

    (b) $f(x) = \cos(x) + \ln|x| + 5e^{5x}$

    (c) $f(x) = -\cos(x) + \ln|x| + \frac{1}{5}e^{5x}$

    (d) $f(x) = -\cos(x) - \ln|x| + \frac{1}{5}e^{5x}$

    (e) None of the above.

19. Which of the following is equal to the arclength of $\sqrt{x}$ from $x = 3$ to $x = 5$?

    (a) $\displaystyle\int_3^5 \sqrt{1 - \frac{1}{4x}}\, dx$

    (b) $\displaystyle\int_3^5 \sqrt{1 + \frac{1}{2\sqrt{x}}}\, dx$

    (c) $\displaystyle\int_3^5 1 + \frac{1}{2\sqrt{x}}\, dx$

    (d) $\displaystyle\int_3^5 \sqrt{x}\, dx$

    (e) None of the above.

20. Calculate $\displaystyle\int_0^1 \frac{e^x}{\sqrt{1+e^x}}\, dx$

(a) $\sqrt{1+e} - \sqrt{2}$

$2\sqrt{1+e} - 2\sqrt{2}$

(c) $2e\sqrt{1+e} - 2\sqrt{2}$

(d) $\dfrac{e}{\sqrt{1+e}}$

(e) None of the above.

u = 1 + exp(x), du = exp(x) dx
int u^(-1/2) du = 2 sqrt(u) + C = 2 sqrt(1 + exp(x)) + C

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21. What are the (standard) domain/range of $y = \text{arccot}(x)$?

Text

(Hint: Remember ▮cot$(x) = \cos(x)/\sin(x)$ has a vertical asymptote every time $\sin(x) = 0$.)

(a) Domain: all $0 < x < \pi$, Range: all y.

(b) Domain: all x, Range: $0 < y < \pi$.

(c) Domain: all x, Range: $-\pi/2 < y < \pi$.

(d) Domain: all x, Range: all y.

(e) None of the above.

22. Which of the following integrals would you use to calculate the area under $y = \text{arcsin}(x)$ between $x = 0$ and $x = 1$? Your choice should be one which you would be able to compute exactly using the techniques from class, without having to rely on approximations.

(a) $\displaystyle\int_0^1 \text{arcsin}(x)\ dx$

(b) $\displaystyle\int_0^{\pi/2} \sin(y)\ dy$

(c) $\displaystyle\int_0^1 \sin(y) - 1\ dy$

(d) $\displaystyle\int_0^{\pi/2} 1 - \sin(y)\ dy$

(e) None of the above.

23. Which of the following is equal to the area between $f(x) = x^3 - 3x$ and $g(x) = x$?
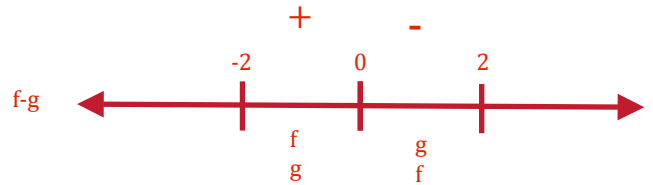
(a) $\int_{-2}^{2} (x^3 - 4x) \, dx$

(b) $\int_{-2}^{0} (x^3 - 4x) \, dx - \int_{0}^{2} (x^3 - 4x) \, dx$

(c) $\int_{-2}^{0} (x^3 - 4x) \, dx + \int_{0}^{2} (x^3 - 4x) \, dx$

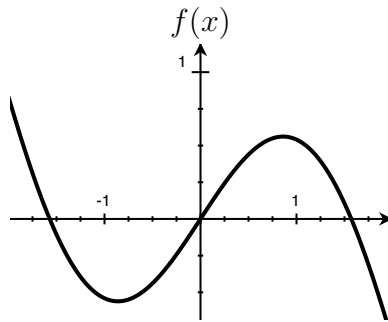(d) $\int_{-\sqrt{3}}^{0} (x^3 - 4x) \, dx - \int_{0}^{\sqrt{3}} (x^3 - 4x) \, dx$

(e) None of the above.

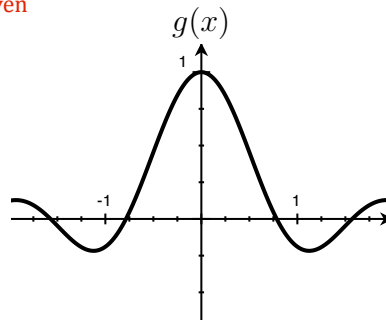solve f=g, so
0=f-g = x^3 - 4x = x(x-2)(x+2)

+   -
-2   0   2

f-g

f   g
g   f

For the next two problems, suppose $f(x)$ and $g(x)$ are the functions pictured below:

odd

$f(x)$

even

$g(x)$

24. Which of the following is *not* an even function? (over the domain shown)

(a) $g(3x)$

(b) $f(g(x))$

(c) $g(x) + 1$.

(d) More than one of the above is not even.

(e) All of (a)-(c) are even.

25. Notice the $f(x) * g(x)$ is an odd function. What can you say about the integral

$$\int_{-1}^{1} f(x) * g(x) \ dx?$$

(a) It's equal to 0.

(b) We don't have enough information to calculate it exactly, but it is equal to

$$2 * \int_{0}^{1} f(x) * g(x) \ dx.$$

(c) We don't have enough information to calculate it exactly, but it is equal to

$$\int_{-1}^{1} f(x) \ dx * \int_{-1}^{1} g(x) \ dx.$$

(d) We don't have enough information to calculate it exactly, but we know it's positive.

(e) There's simply not enough information to make any conclusion.

(For scratch work)