DARTMOUTH COLLEGE MATHEMATICS ACROSS THE CURRICULUM EVALUATION SUMMARY: MATHEMATICS AND HUMANITIES COURSES

March 2000

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Since 1995 faculty involved with the Mathematics Across the Curriculum project at Dartmouth College have developed nine courses linking mathematics topics with humanistic disciplines including art, music, literature, history, and philosophy. In that time fifteen faculty members (eight mathematicians and seven humanists) and 440 students have participated in an experiment that has questioned the conventional approach to undergraduate mathematics.

Math and humanities courses represent one answer the question, "What kind of mathematics does a college student need?" For some students the answer is clear: those who choose a science major need at least some calculus. Interviews with calculus students suggest additionally that some non-science majors pursue calculus for its intellectual beauty and rigor. Most social scientists benefit from knowing statistics. But what about the rest? Calculus and statistics hardly encompass all of mathematics; should they be the only choices? The answer to that question turns on what one believes to be the most effective way to motivate students to learn and use mathematics after college. How can we promote that goal for students who have but a single term's exposure to mathematics? For students who see no relevance in calculus or statistics to their own lives, courses in those topics can be counterproductive, leaving only a distaste for math and a sigh of relief. Math and humanities courses propose an alternative route. Believing that interest in mathematics and confidence in one's abilities to do math are as important to future math success as a panoply of math skills, these courses stimulate interest by exploring new areas of mathematics, using connections with a humanities topic to provide easy entry and relevant applications.

Evaluation results suggest that courses like these fill a gap in the mathematics curriculum. Offering new mathematics in a different pedagogical format attracted both the mathematically sophisticated and the mathematically timid. Despite the challenges of teaching such a bifurcated population, these courses brought new insights and new interest to many. Students enjoyed both the collaborative, applications-based pedagogy and the new mathematical topics they explored. Most emerged more confident about their abilities in mathematics and with a better understanding of its relevance to their own lives. They also acquired a broader and more mature understanding of mathematics as a discipline and its role in other fields. While these courses are still in the prototype stage, we believe they provide a model which can be progressively refined and broadly applied.

Part I of this report describes the pedagogical grounding of the experiment. Part II catalogues the learning goals defined by the participating faculty members. Part III briefly describes the courses and references their course web pages. Part IV provides a demographic profile of the students who chose these courses and explores their motivations in doing so. Part V documents the student response to the courses as revealed in pre-post mathematics surveys and in-depth interviews. Part VI draws conclusions about the role courses like these can play in the college curriculum.

I. Humanities and Mathematics: The Pedagogical Grounding

Alvin White (1993)[1] identifies two themes in the emerging interest in "humanistic mathematics." The first is a desire "to teach mathematics humanistically," by which he means making its pedagogy more student-centered and recognizing that mathematics is socially constructed knowledge. The second theme he describes as "teaching humanistic mathematics," and here he refers to a curricular change which includes the intuitions, value judgments and social forces that shape the discipline's history.

Dartmouth's mathematics and humanities courses embrace both these themes, but more as epiphenomena than impetus. The thrust behind Dartmouth's math and humanities courses is interdisciplinarity, the belief that for many students, mathematics comes to life when it is shown to be relevant to their own interests. This distinction in etiology is important for several reasons. First, it exposes the link among superficially disparate MATC offerings. The motivating role of literature or art in a math and humanities course is the same as physics in the Integrated Mathematics and Physics course: it is the familiar and valued intellectual territory through which mathematics is approached and which, in return, is itself made more complete and interesting by the addition of the mathematical dimension.

Second, the shared interdisciplinary focus points to a shared goal: that students learn real math. Dartmouth's math and humanities courses are not only "about" math, they "are" math. While students study history or art or literature, they also learn group theory or geometry. The insistence that students come to grips with genuine mathematics reflects the belief that understanding math requires doing math, in the same way that understanding literature requires writing. It also reflects the belief that mathematics includes a diversity of topics whose significant ideas are accessible at the introductory level.

The pedagogical and curricular changes White advocates to "humanize" mathematics flow easily from its association with the students' established interests, whether the

[1] White. Alvin. "Preface." *Essays in Humanistic Mathematics*,, Washington, D.C.: The Mathematical Association of America, 1993.

discipline paired with mathematics is "humanistic" or not. Who can read Kepler without appreciating the role of human qualities and social forces in the development of mathematical knowledge? And how much easier to involve students actively when they arrive in class not simply as recipients of new knowledge but as bearers of knowledge—whether of biology, history, or art—ready to make connections with a new field.

II. The Course Goals

Faculty who developed mathematics and humanities courses articulated their goals for the courses in an early focus group and later in individual interviews. They anticipated that these interdisciplinary courses would not only benefit students, but would also be rewarding for themselves and would contribute to a more adventuresome curriculum. It is important to note that while doing mathematics is an important part of the enterprise, acquiring a particular set of mathematical skills is not a goal. For students, they outlined the following goals:

1. To **make mathematics relevant** to undergraduates by
   - exposing the connections between mathematics and other disciplines (including but not limited to science), so that students can use mathematics to ask questions in other disciplines, and vice versa.
   - providing students with a mathematical lens for looking at the world; giving them a mathematical perspective.
   - situating mathematics and science in their historical and cultural contexts.

2. To additionally **increase student interest** in mathematics by making a wider range of mathematics areas (what one mathematician called "the fun stuff") accessible to all undergraduates, not just mathematics majors. Presently students must complete an extended calculus sequence before studying topics such as number theory or group theory or combinatorics. These faculty members believed that mathematical fields like these could be presented to students with modest mathematical backgrounds in a non-trivial way.

3. To **increase students' mathematical confidence** by
   - helping students really understand the mathematics they do, not simply follow rules formulaically.
   - humanizing mathematics, showing that math is done by real people—like themselves.
   - teaching students to communicate mathematical concepts in clear, effective language.

4. To **enhance students' critical and analytic skills** by personally modeling the process of interdisciplinary thinking in the classroom.

5. Through all of the above, to **attract a larger and more diverse student population** to mathematics courses.

For themselves, faculty members had four main goals:

1. To **increase their knowledge and gain a fresh perspective** on their own discipline by working with colleagues outside their field.
2. To **expand their pedagogical skills** by working with colleagues and students from outside their field.
3. To **teach something new** and different.
4. To **connect their own disparate interests** through these courses (e.g., one was a mathematician/composer, another a mathematician/sculptor).

They articulated the following institution-wide goals:

1. To **increase communication and collaboration** across disciplines, especially collaborative teaching between mathematicians and non-mathematicians.
2. To **stimulate greater interest in pedagogy** on campus.
3. To **increase interest in and comfort with mathematics among colleagues** outside mathematics.
4. To **make interdisciplinary mathematics courses and majors a permanent part** of the Dartmouth College curriculum.

III. The Courses

Nine mathematics and humanities courses have been created and taught. Several (Pattern, Late Renaissance Thought, Time) have been offered more than once, sometimes rotating new professors into the collaborating team. Many of these courses have websites with syllabi and supporting materials.

Late Renaissance Thought and the New Universe focuses on the problem of planetary motion and the search for a satisfactory predictive model in the sixteenth and early seventeenth centuries, exploring the interactions between mathematical, scientific, political, philosophical, artistic and magical fields of discourse in the early modern period. Developed and co-taught by a mathematician and an English professor.

Pattern examines the interplay between the art of designing repeat patterns and the mathematics of analyzing those patterns in terms of their symmetries. Through studying and creating works of art—ranging from mandalas to Islamic mosaics to Escher to wallpaper groups—students are introduced to elementary group theory. Developed and co-taught by a mathematician and an artist.

Geometry in Art and Architecture explores the multiplicity of connections between mathematics and art, from proportion to perspective to knots to the influence of numerology on art. Developed and co-taught by a mathematician/artist and an art historian.

Mathematics and Music reveals the mathematical structures and patterns underlying music. Students learn about timbre (through Fourier analysis), scales, melody, rhythm, musical structure. Developed and co-taught by a mathematician/composer and a musician.

How Many Angels? Mathematics and the Infinite links the development of philosophical concepts of the infinite to mathematical understandings. Developed by a mathematician and a philosopher, taught by each at different institutions.

Chaos: Attractive Disorder connects the development of chaos theory in science and mathematics. Developed and taught by a mathematician.

A Matter of Time uses mathematics, literature, and the arts to travel through history, exploring Time as a key concept and reality in the development of Western culture and in our own twentieth century view of ourselves and of the world. Developed and co-taught by a mathematician and a comparative literature professor.

Mathematics and Science Fiction draws on a substantial body of novels and stories that depend on mathematical ideas. Is mathematics is simply a way of mystifying, even intimidating readers or does understanding the underlying mathematics contribute to the total experience of reading a story? This course presents both the mathematics and the literary concepts necessary for an informed reading of the texts. Developed and co-taught by a mathematician and a comparative literature professor.

Renaissance Math in Fiction and Drama explores how scientific developments in Renaissance astronomy were portrayed in literature and drama past and present. Students use Renaissance technology to track the transit of Mars across the sky. Developed and co-taught by a mathematician and a drama professor.[2]

IV. Demographic Profile of Students: If You Build It......Who Will Come?.

A bimodal population. In the end, the name told the story. These courses were not labeled "math for humanists" but "math and humanities" courses, and they attracted competent mathematics students looking for new perspectives on their field as well as humanities students, many eager to meet the College quantitative requirement in a more comfortable context for them than the standard math course. The bimodally distributed population helps explain some of the challenges of designing and teaching these courses. When the courses were offered as first-year writing seminars, they were filled almost completely with strong mathematics students (these were, remember, students who hoped to complete their writing requirement by doing math!). When they were offered for students from all years, the classes included both advanced students and the mathematically timid, but little in between. The table below shows the highest college math course completed by mathematics and humanities students, a good—but not infallible—index of mathematical interest. First year seminars are marked by an asterisk.

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[2] No evaluation data are available for this course, which is offered Winter term, 2000.

Table 1. Highest college math course completed, in percentage of students.

COURSE	No college math	Math 1 or 2 (remedial calculus)	Math 3 (introduction to calculus)	Math 5 (selected topics in math)	Math 8 to 18 (second term to multi-variable calculus)	Math 20 and higher	Total: Math 8 and higher
Pattern S98 n = 31	90.3				3.2	6.5	9.8
Pattern S96 n = 5	80					20	20
Geo. in A&A n = 86	61.6	3.5	12.8	1.2	14	7	21
Kepler S97 n = 34	73.5				11.8	14.7	26.5
Time n = 137	48.9	2.9	7.3	2.9	30.7	7.3	38.0
Math & Music n = 21	19	9.5	14.3		38.1	19	57.1
Kepler W96* n = 6	33.3				66.7		66.7
Science Fict n = 25	32.0				36.0	32.0	68.0
Infinity* n = 16	18.8		12.5		62.5	6.2	68.8
Chaos* n = 15	0.0			6.7	80	13.3	93.3

The table shows that when they were offered as first-year seminars, math and humanities courses drew large numbers of students who had taken math beyond first-term calculus and almost none who had taken lower-level math courses (Math 1, 2, 3, or 5). 67% of "Kepler W96" students, 75% of "Infinity," and 93% of "Chaos" students had taken or were concurrently enrolled in a more advanced math class. The Mathematics and Science Fiction course (not a first-year seminar) also drew a large contingent of strong mathematics students.

With the exception of the science fiction course, however, when math and humanities courses were offered for a general audience, the proportions were reversed: a high percentage had taken no college math. Most of those in these unrestricted classes who *had* taken college math had completed a non-introductory course. Additionally, we know from interview data that some who had taken no college math entered with advanced mathematics credit, increasing the proportion of strong mathematics students in these courses. For example, a third of the students in Kepler S97 with no college math course had AP credits. These tended to be solid high school mathematics students who did not plan a math or science major and thus did not feel calculus was relevant to their interests. Thus the chart above reveals a major contrast, it obscures a third important category of math/humanities students: students with no college math experience but with a strong high school mathematics background. Those with both weak and strong math backgrounds had the same goal in taking a mathematics and humanities course: to meet the Quantitative and Deduction Science requirement. But one group hoped to meet it painlessly, the other hoped to encounter some interesting and challenging mathematics.

Consider these two students, both from the Kepler course offered in Spring 1997:

A drama major: "I had placed into math 17 [multivariable calculus for two-term advanced placement first-year students] or something when I entered, and my freshman fall, I just didn't feel like taking math for some reason, and then I never got back to it. And then, the next year I felt like I had forgotten a lot, and so I was nervous about going back and taking an actual math class, and astronomy related subjects were an interest of mine for a long time. So I thought this might be a good way to do a little bit of the math and get the math credit while also learning about Copernicus and some other philosophers. I thought that was good."

A studio art major: "I'm not qualified to take a real math class at Dartmouth. That's not the reason I got in. When I got here I placed in [mathematics course] zero. Math 1, after one week of that, I decided I just didn't want to have to deal with, you know, hard math problems anymore, so I tried to find a way around it....I only made it through pre calculus in high school. And I didn't even like take a science here, or anything like that."

These two students exemplify the kinds of students mathematics and humanities courses attracted—a population divided between competent and reluctant mathematicians. Interview results revealed the same distinction, and similarly suggested that for many students these courses had appeal beyond checking off distributive requirements. Over half (56%) of the seventy-five students interviewed said they selected the course because they were interested in the topics. A third mentioned fulfilling the quantitative requirement; 21% were there to meet the interdisciplinary requirement. 9% were there because they liked math. Some courses (all the first-year seminars and the science fiction course) were populated almost entirely by strong math students; others drew from both categories. That population difference made them very different courses.

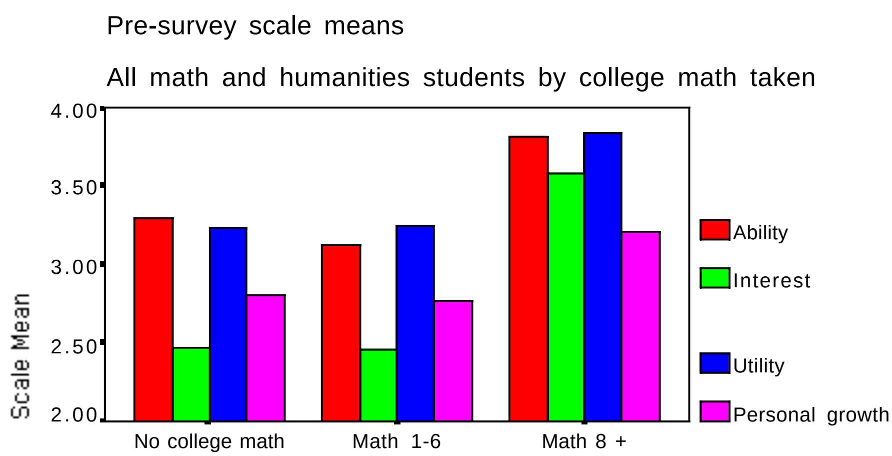
The pre-test responses on the Mathematics Attitude Survey both underscores and qualifies the contrast between these two student populations. The attitude survey is a 35-item, five-point Likert scaled instrument querying beliefs and attitudes about mathematics. It was designed for use in all MATC courses, whether they linked math with humanities or the sciences. Using factor analysis and reliability testing, four scales were constructed: perceived mathematical ABILITY and confidence, INTEREST and enjoyment in mathematics; the belief that mathematics contributes to PERSONAL GROWTH and the belief that mathematics contributes to career success (UTILITY). [3] On these scales, scores for negatively phrased questions have been reversed, so that "5" always represents the desired response and "1" the undesired response. Figure 1 (below) charts the pre-survey means for the four scales for all math and humanities students in the 1997-98 and 1998-99 academic years[4] (N =261). Students who had completed Math 8 were more interested in math and viewed themselves as more capable and more likely the benefit personally and professionally from knowing mathematics than those who had

[3] The ABILITY scale includes Q's 2, 3, 16 (reversed), 22, 27, (reversed), 29 (reversed). The INTEREST scale includes Q's 4 (reversed), 9, 17, 21 (reversed). PERSONAL GROWTH includes Q's 1, 12, 20, 24, 25, 28, 32. The UTILITY scale includes Q's 10, 11, 15, 19 (reversed), 23 (reversed), 30.

[4] Earlier versions of the survey were administered to Kepler W96, Pattern S96, and Infinity F96 and Kepler S97 students. Although scales cannot be constructed for these students, the student responses, by college math course completed, to the eight questions which were consistent through all versions show the same pattern as the 1997-99 students. This suggests that the later classes are representative of the earlier ones, and conclusions drawn from 1997-99 survey data apply to the entire mathematics and humanities population.

taken Math 1-6 or those who had taken no math. [5] The "Math 8+" student means differed significantly on every scale from the "no math" and "Math 1-6" groups; the latter two groups did not differ significantly from each other on any scale.

Figure 1.Pre-survey scale means for three categories of mathematics and humanities students



There are no significant differences between "No math" and "Math 1-6."

"Math 8+" differs significantly from both other groups on all scales.

No college math N = 106
Math 1 - 6 N = 44
Math 8+ N = 111

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[5] One way ANOVA, Tukey's HSD test, $p < .0001$.

The survey confirms the importance of the distinction between those who had and had not taken more advanced college mathematics, but it also corroborates the existence of an important sub-population within the "no college math" category: 14% of those who had taken no college math ranked in the highest third on the "interest" scale; 23% ranked themselves in the highest third in mathematical ability. Thus the "no college math" category includes a minority who had forsaken the standard math sequence but who rated their mathematical abilities and interests as comparable to those who had pursued it. These data are consistent with the interview data noted above and further indicate that the math and humanities courses draw three distinct populations: weak math students who have side-stepped the standard calculus sequence, strong math students who have pursued the standard math sequence, and strong math students who have not chosen the standard math sequence. The challenge for professors was to present mathematics that allowed the more confident and motivated students to elaborate their mathematical interests while simultaneously engaging their more anxious and less interested peers.

Scientists and humanists together. Most courses drew a strong contingent of humanists, usually from a third to 40% of the class. But scientists were also well represented, especially in the first-year seminars. The upper-level courses attracted a broad mix of students from all divisions.

Table 2. Percentage of Math and Humanities students in each major

COURSE	SCIENCE 1 (requires 2 or more terms calculus)	SCIENCE 2 (requires one term calculus)	SOC. SCI. 1 (requires statistics)	SOC. SCI. 2 (requires no statistics)	HUMANITIES	UNDECIDED
Pattern S96		20		20	60	
Pattern S98	3.3	3.3	20	13.3	40	20
Time	9.6	16.9	18.4	8.8	25.0	21.3
Geo. in A&A	11.9	7.1	22.6	11.9	41.7	4.8
Kepler S97	23.5	5.9	11.8	14.7	41.2	2.9
Science Fict.	32.0	4.0	20.0	4.0	20.0	20.0
Kepler W96*	33.3	16.7			33.3	
Math & Music	33.3	14.3	4.8	14.3	33.3	
Infinity*	60	13.3	13.3		6.7	6.7
Chaos*	73.3		13.3			13.3

Men and women, but not always together. While men and women were both attracted to the math and humanities courses, the first-year seminars with a more identifiably "mathematical" theme drew a disproportionately male student body.

Table 3. Percentage of men and women in math and humanities courses.

COURSE	Male	Female
Pattern S96	0	100.0
Math & Music	33.3	66.7
Geo. in A&A	40.7	59.3
Pattern S98	41.9	58.1
Kepler S97	44.1	55.9
Kepler W96*	50	50
Time	50	50
Science Fict	64	36
Infinity*	81.3	18.8
Chaos*	86.7	13.3

A stimulating mix of classes. Students from all years enrolled in these courses. Table 4 documents the wide distribution across class years.

Table 4. Math and Humanities Students by Class

COURSE	FIRST YEAR	SOPHOMORE	JUNIOR	SENIOR
Pattern S96	100			
Science Fict	18.0	16.0	16.0	40.0
Time	43.8	20.4	9.5	24.8
Pattern S98	35.5	29	9.7	25.8
Geo. In A&A	36.9	21.4	16.7	23.8
Kepler S97	26.5	14.7	50	8.8
Math & Music	14.3	57.1	28.6	
Kepler W96*	100			
Infinity*	100			
Chaos*	100			

The implications of diverse student populations. These courses had broad appeal. They drew students from across the disciplines, genders, and the classes. They were attractive to well-prepared math students eager to extend their mathematical competence and to the less confident, who hoped to use other strengths as a bootstrap into mathematics. The aggregation of strong mathematics students in the first-year seminars and the bimodal distribution of mathematical experience in the upper-class courses produced multiple math and humanities "phenotypes." The first-year courses are like hothouse flowers, showing what growth can be achieved when highly motivated and competent mathematics students provide the auspicious conditions. The upper-class courses reveal the challenges of a less uniform and less favorable environment. Both of these examples are instructive. The first-year courses demonstrate the potential of the math-humanities integration for intellectual excitement and discovery. The others remind us of the more common problems of achieving that discovery with an audience that is either uninterested and intimidated or—perhaps the ultimate challenge—one that combines both the mathematically eager and the mathematically reluctant.

V. The Student Response

These courses aimed to increase students' confidence in doing mathematics and to expand their understanding of what math is and what how it might be relevant to their lives. Faculty hoped students would learn to be aware of math in the world around them where they had not perceived it before and that they would find mathematics interesting, perhaps even fun. To evaluate progress toward these goals, students completed a 35-item survey about math attitudes and beliefs on the first and last day of the course. In the early weeks of the term following the course, a random sample of students was drawn and interviewed following a semi-structured protocol. Both survey and interview results suggest that these courses were largely successful in expanding student's understanding, awareness, and appreciation for mathematics.

Mathematics Attitude Survey.

Pre-post changes in the scales. There are a number of ways of looking at survey results. For an overview of course outcomes, let us look first at changes in the four scales from pre- to post-survey. Figure 2 charts the mean change over the term on the four survey scales—ability, interest, career utility, and personal growth—for all math and humanities students for whom we have matched pre- and post-survey results (N = 134).

Figure 2. Pre-post change in the INTEREST, ABILITY, UTILITY, and PERSONAL GROWTH survey scale means for all math and humanities students
Desirable change is represented by positive values, undesirable change by negative values.

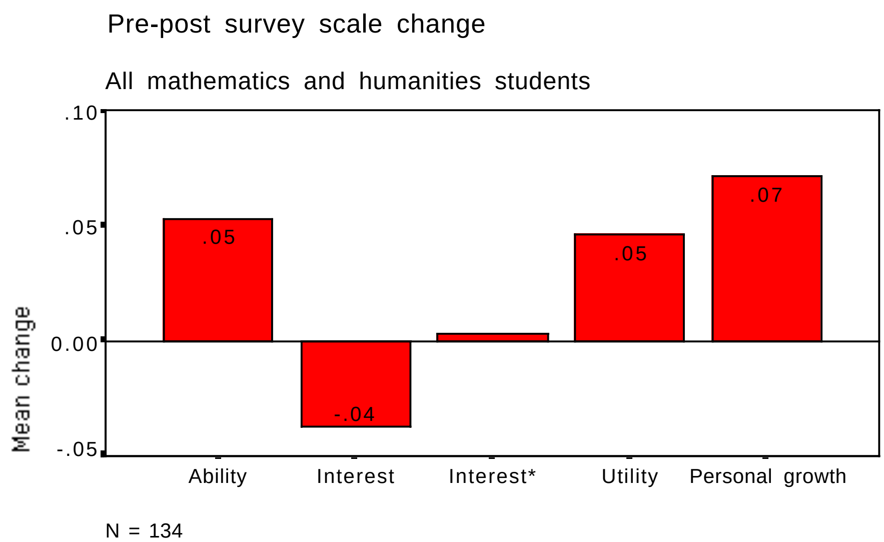


Figure 2 shows that student attitudes changed in the desired direction on the ability, utility, and personal growth scales, indicating that at the conclusion of the course students on average felt more confident about doing math and saw mathematics as more important to their career success and personal development than before. Their interest in mathematics fell, however. While none of these changes is statistically significant, taken together they are informative and interesting. The undesirable change in interest is a somewhat paradoxical outcome, since interest might reasonably be expected to rise as a consequence of increasing confidence and relevance. It may be that the problem is with the measurement.

The category "Interest*" offers an alternative approach to measuring student interest. It could be argued that the word "study" in Item 17—"I want to study more mathematics"—implies a formal school setting, as opposed to a word like "learn" or "acquire," which could be interpreted as happening on the job, while pursuing a hobby, or in any number of other life situations. If students do in fact read "I want to study more mathematics" as "I want to take more mathematics courses in college" they might reasonably "disagree" with #17 at the end of the course, even if their interest in mathematics had been piqued. Completing the mathematics requirement is a principal reason many students enroll in the first place, and many recognize that their college careers are too short to pursue all their interests. When the "interest" scale is recalculated as "interest*" omitting #17, a slight mean *desirable change* from the pre- to post-survey is recorded, instead of an undesirable one. We cannot verify that this interpretation is justified, although it seems a reasonable syllogism. Nor are we prepared to abandon the goal that students be interested *enough* to want to take another math course—beyond that required—in college. But it is important to recognize that increases somewhat short of that goal are also desirable, and the change measured by "interest*" is a legitimate index of success for these courses.

Figures 3 and 4, below, show the pre-post scale mean changes by gender (Figure 3) and by initial level of interest (Figure 4). In both charts, desirable change is indicated by positive values and undesirable change by negative values. Like Figure 2, they include both the original and the recalculated (*) versions of the "interest" scale. These results can only be viewed as suggestive, however, since the difference between men's and women's changes or those of high and low interest students is not statistically significant. Within the sub-populations, the pre- to post- change in the "ability" factor for men, and the change in the "ability" and "utility" factors for high-interest students are statistically significant (no other pre-post changes were significant at the 5% level). Thus the men's increase of .08 in the "ability" scale represents a significant change from the men's pre-survey mean, but the women's gain of .03 does not represent a significant change from their pre-survey mean. However, the men's increase of .08 is not statistically different from the women's pre-post increase of .03 (a difference of that size could have occurred by chance).

Figure 3. Pre-post change in the INTEREST, ABILITY, UTILITY, and PERSONAL GROWTH survey scale means for men and women.
Desirable change is represented by positive values, undesirable change by negative values.

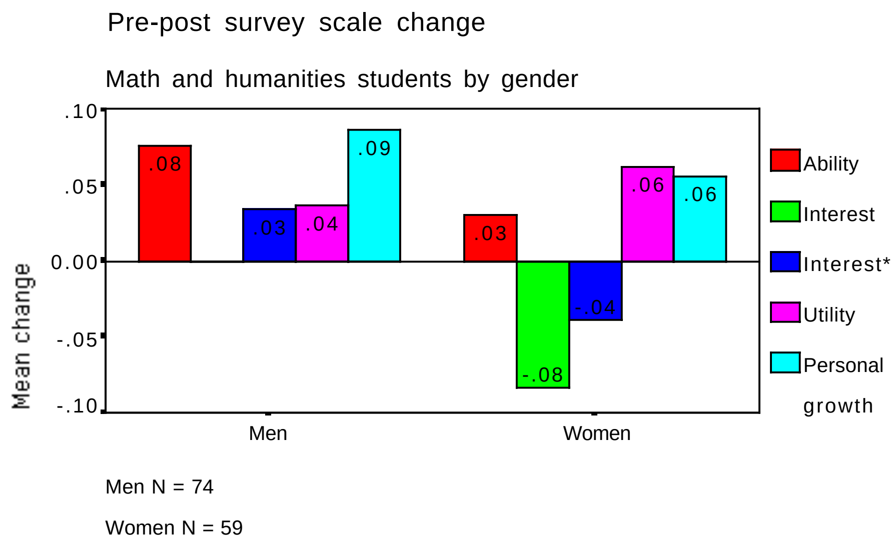
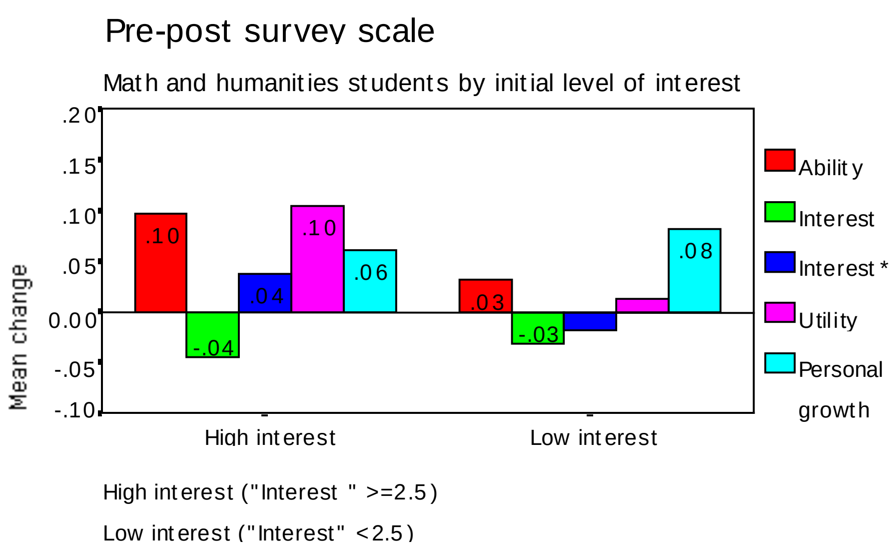


Figure 4. Pre-post change in the INTEREST, ABILITY, UTILITY, and PERSONAL GROWTH survey scale means for students who entered with high interest and low interest.
Desirable change is represented by positive values, undesirable change by negative values.



<u>Pre-post change by number of items.</u> Another rough index of these courses' impact is a simple item count: on how many of the 35 survey items did student attitudes change in the desired direction and in how many in the undesired direction? Table 5 below shows important differences) has been included to allow an absolute comparison which is not a these results for all students for whom matched pre- and post-tests are available, as well as for various sub-categories of students (by gender, level of preparation, level of interest, and major). Because statistical significance is related to sample size, the number of items that increased by .13 (an arbitrary threshold which is "big enough" to reveal dependent on sample size.

	All N=134	Men N = 74	Women N = 59	High Prep (Math 8+) N = 58	Low Prep (<Math 8) N = 76	Sci'nce Major N = 36	Non-Sci'nce Major N = 96	High Inter'st (top 1/3) N = 50	Low Intre'st (low 2/3's) N = 84
Total ♒	21	23	18	28	16	28	16	26	16
Total ♓	14	12	17	7	19	7	19	9	19
♒>.13	4	6	6	9	6	16	4	10	5
♓>.13	2	1	5	4	2	2	4	2	3
Signif ♒	3	2	1	1	2	5	3	2	1
Signif ♓	1	0	2	1	2	0	2	1	1

Total ♒ = total number of items showing change in desired direction
Total ♓ = total number of items showing change in undesired direction
♒ >.13 = total number of items increasing by .13 or more
♓ >.13 = total number of items decreasing by .13 or more
Signif ♒ = total number of items showing statistically significant desirable change
Signif ♓ = total number of items showing statistically significant undesirable change

Overall students made more changes in the desired than undesired direction (21 compared to 14), but it is clear that these courses worked better for students who were more interested and better prepared in mathematics. Students who had completed second term calculus or more or who ranked in the upper third of the sample on the "interest" scale (interest <2.5 on a "1" through "5" scale) on the pre-survey or who were science majors all posted more gains on the survey than did the less prepared, the less interested, or the non-science majors. There is obvious overlap in these categories: half of the "high interest" category were science majors (N=26) and two-thirds had taken Math 8 or higher (N = 35). Three-quarters of the top interest third (N=37) are male. By contrast, 83% of the "low interest" category were non-science majors (N=70) and nearly three-quarters (62 students) had taken no math course beyond first-term calculus. 55% of the "low interest" group are women, 45% are men. But the categories are not identical and the differences among them are informative.

Changes by item. Results so far suggest that most students benefited from these courses, although both the scale changes and the item-count indicate that the gains made by the more mathematically engaged were somewhat greater than those of the less interested. This is not to say that the only differences in survey results were quantitative. Different categories of students responded to the courses in qualitatively different ways, and it is instructive to see how their responses varied. A third way of looking at survey results is to consider the items on which different groups posted the greatest changes in either direction. The table below compares the rank order and the magnitude of pre-post changes by category for all changes in either direction greater than .13. Items where the change was statistically significant are marked with an asterisk (*). Questions which are reverse scored are indicated by an "R" following the question number. The pink-shaded area highlights items on which the population as a whole registered changes in the undesirable direction. The full list of survey items is included below the table for reference.

Table 6. Rank order and magnitude of pre-post survey changes by gender, level of preparation, major, and level of initial interest.

Item	All Rank	All Change	MEN Ch'nge [rank]	WMN Ch'nge [rank]	HIGH PREP Ch'nge [rank]	LOW PREP Ch'nge [rank]	SCI MAJ Ch'nge [rank]	NON SCI MAJ Ch'nge [rank]	HIGH INT Ch'nge [rank]	LOW INT Ch'nge [rank]
12	1*	+.33	+.14 [6]	+.56 [1*]	+.21 [4]	+.42 [1*]	+.25 [8]	+.32 [1*]	+.16 [7]	+.43 [1*]
1	2*	+.27	+.31 [1*]	+.22 [2]	+.41 [1*]	+.16 [5]	+.26 [7]	+.27 [3*]	+.43 [1*]	+.17 [2]
29R	3*	+.20	+.26 [2*]		+.22 [3]	+.18 [3]		+.28 [2*]	+.30 [2*]	+.14 [3]
30	4	+.13		+.22 [3]		+.16 [6]		+.30 [4]	+.14 [9]	
34R	5					+.18 [2]				
22	6					+.17 [4*]	+.19 [10]			+.13 [5]
28	7		+.19 [3]		+.21 [5]		+.39 [4*]		+.22 [4]	
20	8									
5R	9				+.19 [6]		+.50 [1*]		+.24 [3]	
13R	10				+.16 [8]		+.22 [9]			+.14 [4]
10	11				+.17 [7]		+.42 [3*]			
19R	12						+.17 [12]			
4R	13		+.15 [4]		+.16 [9]		+.14 [13]			
14	14			+.19 [4]						
2	15									

15	16		+.15 [5]				+.28 [5]		+.14 [10]	
11	17			+.14 [6]					+.14 [8]	
21	18									
8R	19			+.14 [5]					+.20 [5]	
33	20									
3	21									
27R	22						+.44 [2*]	-.17 [34]		
26	23									
31	24						+.28 [6*]			
35R	25								+.18 [6]	-.13 [33]
16R	26						+.17 [11]			
7R	27				+.26 [2]	-.24 [35]				
32	28				-.17 [33]				-.24 [35]	
6R	29		-.14 [35]							
9	30									
23R	31			-.19 [33]	-.18 [34]					
25	32			-.14 [31]				-.17 [35*]		
24	33			-.22 [35*]		-.24 [34]		-.15 [32]		-.18 [35]
18	34	-.13		-.17 [32]	-.16 [32]		-.19 [34]			-.18 [34]
17*	35*	-.16		-.22 [34*]	-.22 [35]		-.19 [35]	-.16 [33*]	-.32 [35*]	

1. To understand math I sometimes think about my personal experiences.
2. I am good at math.
3. If I work at it, I can do well in math.
4. Most subjects interest me more than mathematics.
5. Mathematics is essentially an accumulation of facts, rules, and formulas to be memorized and used.
6. Good math teachers show students the exact way to answer the questions they'll be tested on.
7. Using a computer makes learning math more complicated than it needs to be.
8. People who are good at math can do math quickly.
9. I enjoy learning new things in math.
10. Math helps me understand the world around me.
11. Mathematics has been an important tool to help me learn other subjects.
12. I have taken some math courses in high school and college that were taught in a very interesting way.
13. For me, mathematics rarely involves exploration, investigation, or experimentation.

14. I like exploring problems using real data and computers.
15. Many situations in the world around me can be modeled mathematically.
16. I often feel like I'm missing something important in math class.
17. I want to study more mathematics.
18. Working in groups helps me learn math.
19. I rarely encounter situations that are mathematical in nature outside school.
20. Doing math helps me understand myself.
21. I try to avoid courses that involve mathematics.
22. When I get stuck on a math problem, I can usually find my way out.
23. Becoming more proficient in math prepares you for the next math class, but that's about all.
24. Writing about mathematics makes it easier to learn.
25. In mathematics you can be creative and discover things for yourself.
26. After I've forgotten all the formulas, I'll still be able to use ideas I've learned in math.
27. I'm never sure my answer is right until I'm given the solution.
28. Doing mathematics raises interesting new questions about the world.
29. Learning mathematics makes me nervous.
30. I often see familiar mathematical concepts in courses outside of math.
31. Doing math helps me think clearly and logically.
32. Mathematical thinking helps me make intelligent decisions about my life.
33. I don't really understand math until I work it out for myself.
34. Expressing scientific concepts in mathematical equations just makes them more confusing.
35. I don't need a good understanding of math to achieve my career goals.

The table highlights a number of points:

• All categories of **students perceived an increased connection between mathematics and their own concerns**—a major goal of these courses. There was the greatest unanimity about Q1, "To understand math I sometimes think about my personal experiences."

• **Students found the courses interesting**. Women, non-science majors, and the low interest and low preparation groups realized their greatest gains on Q12, "I have taken some math courses in high school and college that were taught in a very interesting way." Men, science majors, and the high preparation and high interest groups also made gains on this question, but it ranked much lower for them. This result is open to two interpretations: (1) men and those who have an established interest in math (overlapping categories, to be sure) find this pedagogical approach less interesting than do women and the low interest group or (2) those with a greater interest in math have enjoyed more interesting teaching in the past than those with a low interest, so that this course represents less of a change for them.

• Most students gained in confidence (Q29R, "Learning math makes me nervous"), another goal of these courses. At .12, the women's gains just missed the arbitrary cut-off. It is important to note that those with weaker math backgrounds (the "low preparation" group) made gains on two additional items measuring confidence: Q34R ("Expressing scientific concepts in mathematical equations just makes them more confusing.") and Q22 (When I get stuck on a math problem, I can usually find my way out.").

• **These courses illuminated the relevance of math** to all students, but in somewhat different ways to different groups. The high interest group, along with women, non-science majors, and the low preparation group, all posted gains on Q30 ("I often see familiar mathematical concepts in courses outside of math."). Women and the high interest group also changed in the desired direction on Q11 ("Mathematics has been an important tool to help me learn other subjects."). Men, science majors and the high interest and high preparation groups (who might already have appreciated math's role in other subjects and courses) understood more about how mathematics can model real-world situations, registering desired changes on Q28 ("Doing mathematics raises interesting new questions about the world.") and (with the exception of the high-preparation group) Q15 ("Many situations in the world around me can be modeled mathematically.).

• The stronger math students (science majors and the high preparation and high interest groups), came to **see math as a more open-ended enterprise**, with gains on Q5R ("Mathematics is essentially an accumulation of facts, rules, and formulas to be memorized and used."), and Q13R ("For me, mathematics rarely involves exploration, investigation, or experimentation."). Gains by science majors on Q27R ("I'm never sure my answer is right until I'm given the solution.") and by women and the high interest group on Q8R ("People who are good at math can do math quickly.") also indicate a more mature understanding of mathematics. These students began with a more sophisticated view of mathematics than their less prepared peers; math and humanities courses strengthened those understandings.

• There were **changes in the undesired direction** as well, and many of these were among women, whose declines were most numerous and steepest. It is impossible to know whether this outcome is a response to some aspect of the courses or is instead a reflection of dispositions established long before college. Some losses are easily explained, such as the already-mentioned decline in Q17 ("I want to study more mathematics."). Others are more troubling. All of these courses involved writing about mathematics, but no category of students felt that writing helped them understand math, and four (women, non-science majors, low preparation, and low interest) recorded notable declines on Q24 ("Writing about mathematics makes it easier to learn."). If we examine the results by category of *courses* rather than students, however, we see that students in the first-year seminar, where writing assignments played a much larger role, recorded significant gains on this item. Math and humanities courses also hoped to demonstrate that math could be fun to learn, creative to use, and useful in real life, yet students made undesirable changes in three items addressing those goals: Q9 ("I enjoy learning new things in math."), Q25 ("In mathematics you can be creative and discover things for yourself."), and Q23R ("Becoming more proficient in math prepares you for the next math class, but that's about all.").

<u>Item change in a weighted sample.</u> Finally, it is useful to look at survey results weighted by course, so that each pedagogical/content approach (instead of each student) receives equal weight in the analysis. In the weighted sample of pre-post survey change, students

still record the greatest desirable changes in Q12 (classes were interestingly taught) and Q1 (personal experiences help understand math). Nine of the top twelve ranked items remain the same in the weighted sample, but the order changes. Places 3 - 6 are now occupied by a cluster of items about mathematics' larger relevance: Q28 (math raises interesting questions about the world), Q10 (math helps me understand the world), Q30 (I see math in other courses), Q19R (I see math outside school). In eighth place is Q15 (world can be modeled mathematically). Q29R (math makes me nervous) is in seventh place. At the bottom of the rankings, a similar rearrangement occurs. Ten of the last twelve items are the same in both rankings, but Q16 ("I often feel like I'm missing something important in math class.") moves into last place, suggesting that students are still not always comfortable with the pace and content of classes.

However we look at the survey results, it is clear that students left these courses with a better understanding of how mathematics might be relevant in their own lives and with greater confidence about using it. Whether it was relating math to other subjects encountered in college or to real-life situations, students' appreciation for the ways mathematics might illuminate other areas (and vice versa) was expanded. Many came to understand math less as a formulaic exercise and more as a strategy for exploring real problems. Overwhelmingly, students found the courses interesting. But these data suggest that not all goals are met. Writing is not yet a helpful tool for many students and the value of group work has not been conveyed. The creativity and enjoyment of the discipline are not perceived by many. It is notable that although women recorded some substantial advances, they also posted a large number of undesirable changes.

The survey responses provide a framework defining course outcomes. Exegesis of survey findings comes from the student interviews, where students speak in an unconstrained way about their experiences in the courses, adding immediacy, substance, and some remarkable insights.

The Student Interviews.

A randomly drawn sample of students from each course were interviewed in the subsequent term using a standard interview protocol. Seventy-five students from ten iterations of eight courses were interviewed. The interviews were audio-taped and transcribed verbatim. Only in rare cases did students decline the interview, so we can have confidence in the representativeness of the sample. While students in the first-year writing seminars enjoyed the additional advantages of a small interactive class and mathematically accomplished peers, responses from students in all courses were broadly similar. Overall, 58% of those interviewed said they would recommend their course to another student; 26% more would do so with certain qualifications (e.g., if it were smaller). 16% would not recommend the course the took.

While student endorsement is important (no mathematical gains will be made if students don't enroll), we also recognized that these courses were brand new, using often-unfamiliar pedagogy to teach material from a new perspective. Therefore we wanted to isolate student responses to the novel pedagogical style and mathematical content from general assessments about the value and enjoyment of the course. We asked them to tell us something they learned in the course, and something they learned that would be valuable in other courses or after college. We probed their understandings of mathematics as a discipline and their beliefs about its usefulness and interest. As they described what they had learned, and how they learned it, clear themes emerged.

• **Math applications aid understanding**. Showing how mathematics is used in other fields helps students understand the mathematics better. As one student put it, "I think I learned more completely." Whether a math concept is embodied in a painting or a musical chord, is used as an element in plot development or in fighting forest fires, seeing the mathematics instantiated provides a new avenue by which to achieve comprehension. In the same way that calculus students grasp integrating rates of change better after applying it to real data from a flooding problem, examples from humanistic disciplines can illuminate new mathematical ideas. Listen to these students:

> It was the first time I ever applied math to life. It's neat to be able to look at it and apply it to other things. It was easier for me to grasp the concepts because I could see it. I never really felt the ability to be creative in math, [but this class] gave me the opportunity to work creatively in a more or less not creative subject. It kind of floored me, because I never thought about math and art as having anything to do with each other. [Geometry in Art and Architecture student]
>
> Compared to [other] math courses, it's more interesting because it's not just like they give you a formula and then you give them an answer. It has some kind of applications or something you can hang onto. Some of the math in there, I hadn't seen much of at all. For instance, when we looked at infinity, infinite cardinals and things like that, I had no exposure to that whatsoever. So I felt that I would be able to understand those rather obtuse ideas better in the context of the science fiction stories, so I could see, not exactly practical applications, but just some sort of a demonstration of what they meant. [Mathematics and Science Fiction student]

• Different, non-calculus math stimulates student interest in mathematics. For many college students, calculus *is* higher level math. Many turn away from mathematics in college because they believe calculus is their only option (too often, they are right). These courses introduced students to fields of mathematics usually reserved for those who have completed several terms of calculus. Students were excited to learn that there were whole new worlds of mathematics to discover.

> We talked about advanced applications in math, which you always address slowly. Here we just leapt ahead, and talked about things like the transfinite numbers, and the set theory things that I had never really heard about before. It made me realize that there was a lot out there for people to be researching, because when you take a math class, at the end of the math class, you think okay, we learned *all* of that, but there's quite a bit more. [Mathematics and Science Fiction student]
>
> I kind of think of the math I've done before this, and the math I've done [in this course] very differently. I think they are different branches. What we did here was a lot more

> theoretical, a lot more related to our world. I consider the math I did back in the old days as pure schoolwork. [Mathematics and Science Fiction student]

> [The math] was basically just conceptual. Usually when you do math you don't really sit there and think about why it is the way it is. It's kind of a cut and dried type thing. That's why I took the course, because I wanted to take a course with mathematics background that had more of a philosophical approach to it. So that was the best part of the course for me, to sit there and think about infinity and all that stuff. [A Matter of Time student]

• **Different pedagogical approaches increased student comfort with mathematics.** These courses departed from the conventional blackboard/lecture/problem set format in a number of ways, although all included, at some points, lectures, blackboard exercises, and problem sets. When collaboratively taught, the two instructors were always present, regardless of who might assume principal responsibility for the teaching on that day. Hands-on (or perhaps minds-on) exercises were common. In math and art courses students created their own art and related it to mathematics. In the computer music course students composed electronic music; in the "new universe" course they charted the sky and tracked the planets with historical technology, and in the science fiction course they wrote stories with math at their core. In the classroom, discussion, small group exercises and student presentations were common. Most courses were collaboratively taught, allowing instructors to establish an open-ended dialogue between the two disciplines. For many students, this was their introduction to genuine scholarly dialogue, previously witnessed only in its more restricted literary representation. Frequently, one faculty member could act as the "model student," clever but inexpert in the field. One student observed that it made him feel more comfortable because "you don't feel like you're just working with an expert." All of these teaching strategies helped establish an environment that encouraged student participation and reduced math inhibitions. The students describe their experience:

> I really liked the class activities we did, with the actual hands-on projects, where we duplicated they kinds of things [Renaissance thinkers] would have done, like figuring out why retrograde motion occurs, or by keeping the star journal, and actually going out to track the progress of Mars. That was really interesting. I really got a feel of what it must have been like then, without all the telescopes and mathematical knowledge. [Mathematics and the New Universe student]

> The two of them challenged each other, which was really nice, and they weren't afraid to contradict each other, or to add things to each other's lectures, or to cut one another off. They weren't inhibited by the formality that I think a lot of other professors would be. The collaborative environment they tried to foster with the students was really nice. It felt more like a partnership than "we'll tell you stuff and you learn it." [A Matter of Time student]

• **New approaches and new material help to revive student interest in mathematics....** Some students came to these courses weary of a subject

that seemed more interesting for its challenge than for its content.
Learning different mathematics in a different way restored their interest in the subject. As one student remarked, "I typically thought of math as being cut and dry." Said another, "It definitely made learning the math more fun. It was much more of math as a discovery and a process rather than a means to an answer."
Others observed:

> The chaos part of it made it seem a lot more interesting and colorful. I had thought of math as a black and white sort of thing, because, like pencil and paper, and you write everything down. But just from the pictures and stuff, of like strange attractors, they're like colorful and interesting. I don't know, I guess it made it colorful, sort of, having it put to use." [Chaos student]

> I guess [my interest in math] died off when I took multi-variable calculus and differential equations. Taking this course kind of renewed my passion for math again. I kind of realized that there is other stuff other than the dry stuff then I've been doing in class. This is actually very interesting. [Mathematics and Science Fiction student]

• **...and restored students' confidence in their mathematical abilities.** Others who had lost confidence in high school or college found a fresh start in courses that took up different material in a different way.

> I think its not one big thing that I learned. The great thing about courses like Pattern for me [was] that it did give me confidence about math again, but in a different way....I just learned its place in the universe or whatever. That it's always connected and that I can do it, that I can succeed in math. [Pattern student]

> It put me a little more at ease with the humanities and the mathematics. It's weird to think that I was a really good math student in high school, and I'm intimidated now by math, and it's kind of sad to me. But when it was presented with the humanities and vice versa, it was more palatable. I thought it was a very good class. [A Matter of Time student]

• **These courses gave them a new understanding of what mathematics is...** Studying different mathematics in a different way gave many students a new view of the subject itself. As the comments below illustrate, most students enter with a narrow definition of mathematics, derived from years of problem sets in one or two sub-fields. Few are aware of the breadth of the subject, its level of abstraction, or its applications in the non-sciences. The fact that mathematics was an expanding field, as uncertain and mysterious at its boundaries as other sciences, was a revelation to many.

> I had never thought of math quite the way it was presented. I always thought of it as a tool that you used for designing a building or an airplane, [but] it almost came across as a type of religion. It opened my eyes. [Infinity student]

> I typically thought of math as being cut and dry, that there are people who have new theories and they're proven or disproven and math kind of grows. But, it turns out that

> there's really a lot of arguments that nobody really knows the real answer to at the basis of mathematics. [Infinity student]

> Well, this particular course was [about a topic] which was like I guess, scientists were still studying it, it was still....a developing course. Which made it more interesting in a sense, compared to all the other math courses. In other math courses, the facts were there, and you'd just like study it, learn it. But in this course, there were certain areas where one could wonder over what happened. Nobody was really sure. [Chaos student]

•...and a new way of looking at other subjects and the world. The realization that mathematics is imbedded in human enterprise from sonatas to skyscrapers to science fiction not only changed the way students looked at math, it changed the way they looked at the world. Buildings, wallpapers, paintings, rock songs, and novels assumed a new dimension for many.

> It's helped me to see the world through a more mathematical eye, where I would normally just see it as just everyday things. It actually made me take a second look at the world. [Chaos student]

> I can't recite to you any facts I learned because it wasn't that kind of class. If I thought about it, I could probably talk a little about Einstein or whatever, but I really just thought about the world differently. It was more like they gave me something and I selected it and just internalized it. [A Matter of Time student]

> And I guess that what I found ultimately was that the math component of music is what ties it to us as humans. The fact that it's regular, that we don't understand things that are completely random. And that we need to connect it to earth, we need to have some type of pattern. [Mathematics and Music student]

> Before, my concept of art was that the artist has the idea in his mind, and he creates his work on paper. But it was really incredible to me to see how very mathematical some of these painters thought, and how they created their work based on something other than imagination. It was very mathematical. I could see that they had thought very analytically about their painting, and that blew me away. I was like, wow, you know, math really is applicable to things other than just equations. [Geometry in Art and Architecture student]

• Math and humanities courses modeled interdisciplinary thinking. Perhaps as important as their advances in mathematical learning and interest was the students' introduction to interdisciplinary thinking. The ability to connect ideas and methods from different fields is increasingly recognized as important for future success in all areas, but it is rarely addressed directly by the curriculum. These courses did so in a dramatic way, linking mathematics to subjects where the overlap is not obvious. Professors consciously modeled the kind of creative thinking required to expose these connections. As students discovered, if you can build a bridge between subjects this different form one another, you are ably prepared to make the interdisciplinary leap in other instances. See how the students below answered the question, "Can you tell me something you learned in the course?"

> [What I learned was] the interdisciplinary approach—just the knowing how to integrate material that doesn't necessarily at the beginning seem like it would fit together. Finding ways, exploring and being creative in math which you wouldn't, you'd say creative and math don't go together. You're not creative in math. But you can be. And learning that you can be. And learning that when someone says, 'Can you do these two things?' and you say,' no' you probably can. You just need to figure out how. [Pattern student]

> Something I learned in the course? [I learned] how to think more broadly, and look at things in a less than mainstream way, kind of off the beaten path, and just took a different approach to ordinary things. [Infinity student]

> I learned [how to] look at things from two different angles, and see how different aspects of a subject can fit into another subject that you would never relate before. I think that's really valuable, because it makes me more open minded, and more interested in looking at things from a different angle. [Geometry in Art and Architecture student]

> [What I learned was] just being able to look at whatever I do from different angles now, generally not seeing math for just being math but seeing its effects on humankind and conversely seeing the effects of literature on math, and how it's interpreted. [A Matter of Time student]

Like the survey results, the interviews suggest that mathematics and humanities courses engaged and stimulated many students. They came to appreciate that mathematics had applications in subjects beyond the sciences, and that understanding math could deepen their understanding of humanistic subjects. Learning new mathematics in a new way increased their interest in math and their confidence in approaching it. Their understanding of mathematics as a discipline was revised and expanded, their view of the world was enlarged, and their analytic abilities were enhanced as they witnessed and practiced making connections between math and the humanities. Most would agree with the assessment of this student, who had "hit the wall" in high school calculus:

> The world is not divided into little groups. It's all very interrelated. So to have an education system that draws clear distinctions between math and science and English, it's very unrealistic. I don't think that that's the way for the majority of people to learn. I think the reason a lot of people shy away from math or science is because it's not like a tangible subject that you can relate to different aspects of your life. Which is very false.

But not all students were pleased with these exercises in boundary blurring, and not all courses were successful in every way. Some of the student dissatisfactions were not relevant to the interdisciplinary or mathematical nature of the courses: complaints about classes that are too large or professors whose approach is not congenial are independent of the motivating thrust of these courses. But some were pertinent. Student comments, combined with the evaluator's classroom observations and interviews with faculty highlight some of the challenges of developing and teaching these innovative courses. The next section discusses some of the lessons learned in four years of teaching mathematics and humanities courses.

VI. Challenges and Recommendations

After ten course iterations with fifteen different faculty members, the challenges of teaching mathematics and humanities courses have become clear, as have some strategies for meeting those challenges. First, working in a collaborative teaching team entails a different style of preparation and teaching. Preparing, lecturing, testing, grading—all must be accomplished differently. The comfortable privacy of the solitary instructor preparing the lecture to be presented to an audience no more discerning than its cleverest undergraduate is replaced by a very public colloquy with peers. Second, the interdisciplinary dictate itself requires a new approach. Even when these courses are taught by a single professor, the rationale for interdisciplinary learning conduces to a different organization and presentation of material. Third, teaching a heterogeneous population poses enormous challenges, especially when the variation encompasses not only different levels of preparation in the two subjects but also different levels of motivation and mathematical comfort. These courses carried pre-requisites in neither subject, so that a math and music course, for example, was open to students with backgrounds in neither math nor music, making the concept of bootstrapping one's way into math through a humanity a very generalized activity at best. The challenges are sizable, and the possibilities for failure are numerous, but these courses have demonstrated (sometimes by the painful but productive trial and error method) that there are also paths to success. The paragraphs below summarize the major findings.

• Exploring the points of intersection between the two disciplines is the rationale for these courses and the measure of their success. Interdisciplinary teaching is not an exercise in parallel play, a sort of "one from column A, one from column B" approach to learning where each instructor presents half a course's worth of each subject. The goal of these courses is to illuminate the connections between two disciplines and to demonstrate the modes of thinking that facilitate the effort. As the student citations above show, the intellectual growth and excitement come from learning how to make those connections. When it doesn't happen, they notice. As one student commented of his experience, "The two of them [the disciplines] just never came together. They were coherent, but they were coherent as separate entities."

The interdisciplinary rationale has several entailments. First, it requires looking at one's discipline in a fresh way, looking less for the most important concepts than for productive points of linkage, those that can form one end of a bridge to the other discipline. Second, it enjoins instructors to model interdisciplinary thinking. It is not "cheating" to guide students through the process of connecting disparate fields. Students already have the option of taking two courses and drawing connections between them independently, and many do this. Interdisciplinary courses exist to introduce students to the analytic practice of finding patterns and connections where they are not obvious, so that they may more readily generalize the technique to other areas. Third, it suggests that professors be transparent about their goals and strategies. Students also have well-developed concepts of a "course" and how it should proceed. Interdisciplinary courses usually break those rules, and they can leave students bewildered about their direction and purpose. Sharing with students why material and activities are relevant and important is not just reassuring, it makes students partners in the enterprise.

• **Teaching collaborative, interdisciplinary courses is more work than teaching alone.** It is also more work than instructors anticipate. (It is also, professors report, more fun.) This follows from the point above. It might seem counterintuitive that being responsible for half the material is more work than being responsible for all; this is because commanding and presenting half the material is only the starting point in an interdisciplinary course. The structure of these courses flows from the points of disciplinary intersection, calling for a development of material and ideas that is likely to be different from an instructor's other courses. While content is always important (one must have something to think *with*), the process of linking two fields is paramount—and new.

• **Collaboration is a full spectrum activity.** Corollary: the more extensive and complete the faculty collaboration, the better the course. Collaboration begins at the beginning, when the main themes and general outlines are chosen, and should extend to all areas of the course: course planning, classroom presentation, testing, grading. Pedagogical issues usually arise when individual teaching styles and different disciplines are brought together; if they don't, the planning discussions are probably not thorough enough. Discussing how to address different topics and exploring teaching strategies is a valuable part of course planning.

One faculty participant described collaborative teaching as "like marriage;" one of the most successful MATC math and humanities courses was taught by a married couple. The marriage metaphor highlights the qualities of trust, respect and sharing that make for a productive collaborative relationship. At the very least, both faculty members should be present at all class meetings, ready to contribute whether it's "their time" or not (and their contribution should be welcomed). Students frequently recommend that both professors should have at least student-level command of both subjects. Their reasoning: if we can learn it in ten weeks, why shouldn't they? Their suggestion is compelling. How can instructors negotiate the interstices of their two disciplines with knowledge of only one? And what message do they convey about the value of interdisciplinary learning if they aren't committed to doing any? Finally, collaborating instructors should be clear and agreed about assignments and grading standards (here the parental metaphor is more appropriate). Because this approach is new to students also, they are sometimes uncertain how to execute assignments ("What does she mean when she says to use math when I explain this point by Plato?"). They are always (and justifiably) distressed when grading standards are not uniform.

• **Don't be afraid to teach math.** Despite the great range of mathematical interest and abilities represented in this student population, almost every student interviewed was there to learn math. Few who feared math were there to avoid math; they were there to surmount their fear. When courses failed to challenge students by presenting math that was too easy or by stressing the humanities portion at the expense of the math , even the weak math students complained. Few students like feeling handicapped in math, and they seek the opportunity to demonstrate that they can learn new math. Engaging such a heterogeneous group is not easy. One problem is to find interesting and challenging topics that can be addressed by students with modest math backgrounds. Another is to devise a pedagogical approach that will render those topics accessible. These courses typically approached the topic conceptually and included both applications and problem sets to advance comprehension. In some cases the most mathematically accomplished students must be satisfied to make advances on other fronts. (In interviews, most recognized and accepted that the challenges in these courses for them could not come from learning new mathematics.) A review of the syllabi for these courses shows how topics can be linked to illustrative humanities areas, making them accessible to a broad range of students.

VI. Summary

Dartmouth's pioneering mathematics and humanities courses have surmounted the daunting challenge of making mathematics accessible to a population ranging in preparation from high school algebra to differential equations, and in interest from the terrified to the wildly enthusiastic. As students perceive that mathematics can illuminate humanistic subjects (and vice versa), they also appreciate that mathematics can be a creative undertaking which is useful in all fields. Surveys and interviews reveal increased confidence and a new openness to mathematical thinking. Perhaps as important as their mathematical gains is their exposure to interdisciplinary dialogue. As professors actively construct a conversation between highly disparate disciplines, they model the kind of intellectual agility needed to negotiate an increasingly compartmentalized world. As one student concluded of her experience, "This course makes you think, and that's not a cliche."

These courses fill a gap in the mathematics curriculum, offering different rewards to different categories of students. For the mathematically timid, they promise a fresh start, an opportunity to learn different math in a different way. Students who have not thrived in conventional courses, or whose confidence was not bolstered in that format, can establish a new mathematical base from which to proceed. For the mathematically advanced, these courses are a lagniappe, an unexpected bonus of applications and insights to add to their already well-appointed mathematical kits. One previously under-served subset of the advanced group are those who arrive at Dartmouth with advanced placement mathematics credit but no desire (or need) to pursue calculus. These students must meet the College's quantitative and deduction sciences requirement, but were previously limited to advanced calculus courses or statistics. The math and humanities courses provide sustenance for their mathematical interests.

The MATC math and humanities courses stand as a prototype—not fully refined, but highly functional. Interdisciplinary mathematics is a novel undertaking for faculty and students alike, and both have to readjust their expectations and practices. As collective knowledge about developing and teaching these courses continues to grow, student outcomes should be stronger.

Dartmouth College Mathematics Across the Curriculum Survey

This course is part of the Mathematics Across the Curriculum project, a five-year National Science Foundation initiative to introduce innovative mathematics instruction into many different subjects. This survey of student attitudes is part of a continuing effort to measure the impact of the project on the College as a whole. All students in MATC courses are invited to complete the survey. Please answer honestly and thoughtfully and remember, there are no right or wrong answers. Your responses are very important to the research and will be treated with great respect and confidentiality. Thank you.

PLEASE FILL IN BUBBLES COMPLETELY WITH A #2 PENCIL: Correct ● ○ Incorrect ✓ ☒

	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
1. To understand math I sometimes think about my personal experiences.	1	2	3	4	5
2. I am good at math.	1	2	3	4	5
3. If I work at it, I can do well in math.	1	2	3	4	5
4. Most subjects interest me more than mathematics.	1	2	3	4	5
5. Mathematics is essentially an accumulation of facts, rules, and formulas to be memorized and used.	1	2	3	4	5
6. Good math teachers show students the exact way to answer the questions they'll be tested on.	1	2	3	4	5
7. Using a computer makes learning math more complicated than it needs to be.	1	2	3	4	5
8. People who are good at math can do math quickly.	1	2	3	4	5
9. I enjoy learning new things in math.	1	2	3	4	5
10. Math helps me understand the world around me.	1	2	3	4	5
11. Mathematics has been an important tool to help me learn other subjects.	1	2	3	4	5
12. I have taken some math courses in high school and college that were taught in a very interesting way.	1	2	3	4	5
13. For me, mathematics rarely involves exploration, investigation, or experimentation.	1	2	3	4	5
14. I like exploring problems using real data and computers.	1	2	3	4	5
15. Many situations in the world around me can be modeled mathematically.	1	2	3	4	5
16. I often feel like I'm missing something important in math class.	1	2	3	4	5
17. I want to study more mathematics.	1	2	3	4	5
18. Working in groups helps me learn math.	1	2	3	4	5
19. I rarely encounter situations that are mathematical in nature outside school.	1	2	3	4	5
20. Doing math helps me understand myself.	1	2	3	4	5
21. I try to avoid courses that involve mathematics.	1	2	3	4	5
22. When I get stuck on a math problem, I can usually find my way out.	1	2	3	4	5
23. Becoming more proficient in math prepares you for the next math class, but that's about all.	1	2	3	4	5
24. Writing about mathematics makes it easier to learn.	1	2	3	4	5
25. In mathematics you can be creative and discover things for yourself.	1	2	3	4	5
26. After I've forgotten all the formulas, I'll still be able to use ideas I've learned in math.	1	2	3	4	5
27. I'm never sure my answer is right until I'm given the solution.	1	2	3	4	5
28. Doing mathematics raises interesting new questions about the world.	1	2	3	4	5
29. Learning mathematics makes me nervous.	1	2	3	4	5
30. I often see familiar mathematical concepts in courses outside of math.	1	2	3	4	5
31. Doing math helps me think clearly and logically.	1	2	3	4	5
32. Mathematical thinking helps me make intelligent decisions about my life.	1	2	3	4	5
33. I don't really understand math until I work it out for myself.	1	2	3	4	5
34. Expressing scientific concepts in mathematical equations just makes them more confusing.	1	2	3	4	5
35. I don't need a good understanding of math to achieve my career goals.	1	2	3	4	5

Demographics

36. Please fill in the last six digits of your Social Security Number. This information enables us to match surveys from the beginning and end of the course. It does not enable us to identify you.

0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

37. What year are you?

- First Year
- Sophomore
- Junior
- Senior
- Graduate Student
- Other

38. With which of the following groups do you self-identify? (Mark all that apply)

- African-American/Black
- Asian/Pacific Islander
- Latino/Hispanic
- Native American
- Caucasian/White
- Other

39. What sex are you?

- Male
- Female

40. How old are you?

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9

41. What was your average grade in high school, overall and in your math courses?

Overall		Math Courses
○	A	○
○	A-	○
○	B+	○
○	B	○
○	B-/C+	○
○	C or below	○

42. What is your average grade in college, overall and in your math courses? (First term, first-year students leave blank)

Overall		Math Courses
○	A	○
○	A-	○
○	B+	○
○	B	○
○	B-/C+	○
○	C or below	○

43. What is your college major? (Mark all that apply)

Humanities

- Art/Art History
- Classics
- Comparative Literature
- Drama
- English
- Film Studies
- Foreign Language
- Music
- Philosophy
- Religion

Sciences

- Biology
- Chemistry
- Computer Sciences
- Earth Sciences
- Engineering
- Mathematics
- Mathematics and the Social Sciences
- Physics and Astronomy

Social Sciences

- Anthropology
- African/African Amer. Studies
- Asian Studies
- Economics
- Education
- Geography
- Government
- History
- Latin Amer. & Caribbean Studies
- Linguistics and Cognitive Science
- Psychology
- Sociology
- Women's Studies
- Native American Studies

- *Undecided*

44. Mathematics material you covered in High School: (Mark all that apply)

- Algebra
- Geometry
- Advanced Algebra
- Trigonometry
- Statistics
- Finite Mathematics
- Pre-Calculus
- Calculus

45. Mathematics courses you have taken in college: (Mark all that apply, including courses you are presently enrolled in)

- Math 1-Calculus w/Algebra
- Math 2-Calculus w/Algebra & Trig
- Math 3-Intro to Calculus
- Math 5-Exploring Mathematics 'Chance' 'Pattern'
- Math 6-Intro to Finite Mathematics
- Math 7-First Year Seminar
- Math 8-Calculus & Linear Algebra
- Math 9-Calculus & Linear Algebra, Honors
- Math 10-Elementary Statistics
- Math 11-Calculus; Advanced Placement
- Math 12-Calculus; Advanced Placement-Honors
- Math 13-Multivariable Calculus
- Math 14-Multivariable Calculus, Honors
- Math 15-Integrated Math & Physical Science
- Math 16-Linear Programming
- Math 17-Multiv. Calc.; 2-term adv. placement
- Math18-Multiv. Calc.; 2-term adv. placement, Honors
- Math 20-Discrete Probability
- Math 22-Linear Algebra
- Math 23-Differential Equations
- Math 24-Linear Algebra, Honors
- Higher Level Mathematics Course(s)

46. Course in which you completed this survey:

		Course #	
Biology	1	0	0
Earth Sciences	2	1	1
Economics	3	2	2
Geography	4	3	3
Government	5	4	4
Mathematics	6	5	5
MASS	7	6	6
Physics	8	7	7
Psychology	9	8	8
Sociology	10	9	9
College Course	11		

Term

- Fall
- Winter
- Spring
- Summer

Year

- 1997
- 1998
- 1999
- 2000

Student Interview Protocol

As you may know, the Math Across the Curriculum Project includes many different kinds of courses, but all of them feature some new way of approaching their subject. Our evaluation uses many different methods to try to determine whether these new approaches are effective, but one of the most important measures is the students' response to them. The goal of these interviews is to add the student voice to our understanding of these new courses. Everything you say will be treated with the greatest confidentiality.

1. Why did you decide to take this course?

2. Can you tell me a little bit about your math background?
   What is your intended major?

3. How was this course different from other courses you have taken at Dartmouth? Elsewhere? Did those differences change what you learned? How you learned it?

4. How did you feel about having more than one professor in the classroom? How did it affect your learning?
   Prompt: Did it facilitate / inhibit your learning?

5. Now I'd like to ask you about the integration of material from different disciplines in this course.

   • What do you think were the goals of this course?
   • Was it successful?
   • Did the course hang together?
   • Did having several disciplines integrated affect your learning of material from either--or both--disciplines?

6. Can you tell me something you learned in this course?
   Prompt: Why did you pick this to tell me about?

7. Can you tell me something you learned in this course that you believe will be useful in other subjects or after college?

8. Can you give me three examples of how mathematics is related to other fields or disciplines?

9. Did this course change your perception of how mathematics is related to other subjects?

10. How would you define mathematics?
    - Did taking this course change the way you think about mathematics?
    - Did it change your level of interest in mathematics?

11. Did taking this course change the way you think about the other subject? How?

12. What do you think are the strengths of this course?

13. What do you think are the weaknesses of this course?

14. Would you recommend this course to another student? Why or why not? What you would tell them about the course?

    - Is there anything else you would like to tell me about this course?