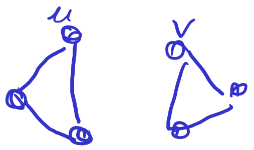
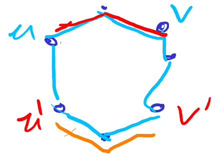
Prove or disprove: If every vertex of a simple graph G has degree 2, then G is a cycle.



This is not true, because here is a counter-example. There is no path between vertices u and v.

However, if the graph is connected, and each vertex has degree 2, then G is a cycle.
This is true, whenever the graph has a vertex v and the whole graph is a path from v to itself that goes through all the edges, and that touches every vertex only once.
Since the graph is connected, there is, for every pair of vertices $\{u,v\}$, a path from u to v.
Call this path P, and make a copy of G without the edges of P.
In that copy, take the vertex that is next to u and call it u'. Do the same with the vertex next to v and call it v'. We know they exist, since u and v have degree 2.
Since the graph G is connected, there exists a path between u' and v'.
The path that goes from u to v + the edge vv' + the path from v' to u' + the edge $u'u$ is a cycle.



The proof that if G is connected and every vertex has degree 2, it is a cycle can also be done very easily by induction, with base case the complete graph with three vertices.