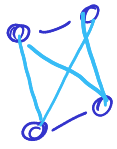


1.2.20 Let v be a cut-vertex of a simple graph G . Prove that $\bar{G}-v$ is connected.



If G is disconnected, then its complement is connected (by homework 1). Also, $\bar{G}-v$ is connected if v is not an isolated vertex, because all the vertices in its components are connected to all the vertices in the other components. v cannot be an isolated vertex, since it is a cut-vertex.

If G is connected, $G-v$ is disconnected, since v is a cut-vertex. That means that the complement of $G-v$ is connected. However, $\overline{G-v} = \bar{G}-v$. This is because an edge uv will disappear from G and its complement after removing v , and an edge uw with u and w not equal to v will not be impacted by removing v . This proves that $\bar{G}-v$ is connected.

