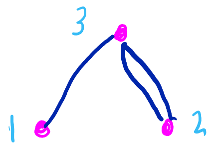
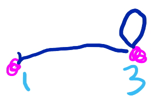
1.3.14 Prove that every simple graph with at least two vertices has two vertices of equal degree?

Assume the graph G has n vertices. The possibilities for the degree are $\{0, \dots, n-1\}$. However, G cannot have one vertex of degree 0 and one vertex of degree $n-1$, because these two vertices would need to be adjacent to satisfy degree $n-1$.
Using the pigeonhole principle, we must pick the n degrees (one for each vertex), from a set of $n-1$ answers (since we cannot have both 0 and $n-1$). Hence, one degree must be repeated.

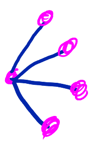
It is not true for graphs with no loops:



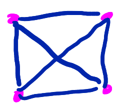
Is it true for graphs without multiple edges? No.



1.3.15 a) For each $k \geq 1$, determine the smallest n such that there is a simple k-regular graph with n vertices.



I will prove that $n = k+1$.
1) Since the graph is simple, the maximal degree of a vertex in a graph with n vertices is $n-1$. To have degree k, we need at least $k+1$ vertices.
2) The complete graph with $k+1$ vertices has degree k, for all vertices, so it is k-regular.



True or False: If u and v are the only vertices of odd degree in G, then there is a u,v-path in G.
Answer: True