Notes on proof writing

Tip #1: Be honest with what you know.

Tip #2: Use vocabulary/notations you know. Use words and English sentences.

Tip #3: When finishing a proof, make sure everything is useful.

Proof techniques:
- Direct proof (example: degree-sum in notes 04/10)
- Contraposition: A => B is equivalent to (not B => not A)
- By contradiction (see below)
- Induction (see next page)
- Bijection : Helpful for counting problems, we pair what we want to count with something we know how to count (permutations, sets, etc.)
- Construction : example is uv-path in notes 04/13
- Algorithmic : Please check your algorithm terminates and does the right thing - Examples are the bipartite subgraph in notes from 04/13 and the graphic sequence in notes from 04/15
- Examples and counter-examples : Examples can be proofs for existence problems. Counter-examples can be used to prove a theorem wrong.

Example: proof by contradiction.
Proving that $\sqrt{2}$ is irrational.
Assume instead that $\sqrt{2}$ is rational, so $\sqrt{2}=\frac{a}{b}$, a,b integers, $\gcd(a,b)=1$.
Then, $2b^2=a^2$. Then, a^2 is even, and a is even as well. Let's write $a = 2c$, c an integer.
That means that $2b^2=4c^2$, of $b^2=2c^2$, which means b is also even.
So a and b are not coprime.

Proof by induction:
In both cases, we start with the smallest example (base case), often $n=1$ or $n=0$.

Induction hypothesis:
- Regular induction: For a given value of n, $X(n)$ is true.
- Strong induction: For a given value of n, $X(j)$ is true, for all $1 \leq j \leq n$.

Induction step:
- Regular induction: We need to prove $X(n+1)$ is true, using that $X(n)$ and $X(1)$ are true.
- Strong induction: We need to prove $X(n+1)$ is true, using that $X(j)$ is true for all j between 1 and n.

Examples
Regular induction: Minimum number of components on 04/06.
Stong induction: Uv-walks and uv-paths, on 04/06

Proving an "if and only if" statement: You must prove that the condition is both necessary and sufficient. In "A <=> B", condition B is

$\boxed{(\Rightarrow)}$ necessary
$\boxed{(\Leftarrow)}$ sufficient

to prove condition A. If you cannot remember the vocabulary, remember at least that you have to prove both and draw the arrows.