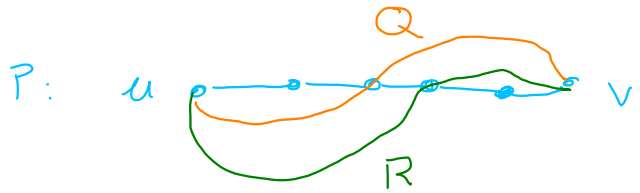
4.2.4 True or False: If P is a uv-path in a 2-connected graph, then there is a uv-path Q that is internally disjoint from P.



The statement is false, as shown by the counter-example.

4.2.24 Let G be a k-connected graph, and let S and T be disjoint subsets of V with size at least k. Prove that G has k pairwise disjoint ST-paths.

We know that

$$\lambda(u,v) \leq \kappa(u,v) \geq \kappa(G) \geq k$$

Case 1: If u,v are not adjacent, then $\lambda(u,v) = \kappa(u,v) \geq k$.

Internally disjoint ⇒ pairwise disjoint

Case #1: There exists u in S and v in T that are not adjacent. By Menger's theorem, there are at least $\kappa(u,v)$ internally disjoint paths, so there are at least k uv-paths that are disjoint. Also, every u,v-path is an S-T path, so there are at least k ST-paths.

Case #2: Every vertex of S is adjacent to every vertex of T. The number of edges between S and T is $|S||T| \geq k^2$. Edges are internally disjoint paths.