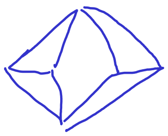
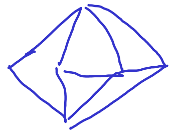
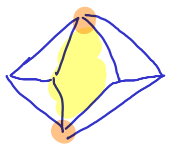
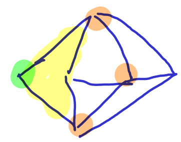
5.3.18 Compute the chromatic polynomial of these 2 graphs.

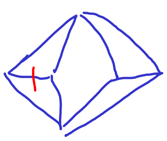
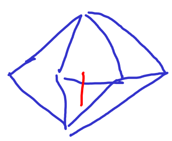
1 — Show they have the same chromatic polynomial.

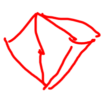
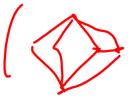
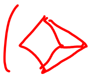
 

They are not isomorphic, because only one has a vertex of degree 2.

However, using contraction deletion with different edges, we get the two same graphs:

$\chi($  $; k) - \chi($  $; k)$ $\qquad$ $\chi($  $; k) - \chi($  $; k)$

So they have the same chromatic polynomial.

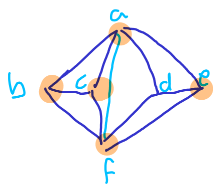
2— Compute the chromatic polynomial of any of these two graphs.

Observation

The deletion—contraction recurrence for the chromatic polynomial implies the following:

$$\chi(G)=\chi(G-e)-\chi(G.e) \Rightarrow \chi(G-e)=\chi(G)+\chi(G.e)$$

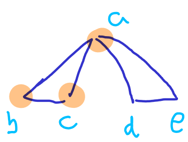
Applying it to this problem, we add the light blue (vertical) edge.



It has a simplicial elimination orderng d,e,c,b,f,a.

a	f	b	c	e	d
0	1	2	3	2	3

$$\chi(G;\ k) = k(k-1)(k-2)^2(k-3)^2$$



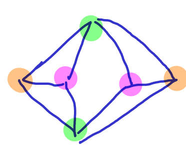
It has a simplicial elimination orderng d,e,c,b,a.

a	b	c	d	e
0	1	2	1	2

$$\chi(G.e;k) = k(k-1)^2(k-2)^2$$

$$\chi(G-e;k) = k(k-1)(k-2)^2((k-3)^2+(k-1))$$

G-e



And that gives the chromatic polynomial of the two graphs above.