Math 13 - Second Hour Exam, Winter 2002

Part I: Multiple choice. Each problem is worth 5 points.

1. The integral \( \int \int_{R} \frac{\sqrt{y}}{x} \ dx \ dy \), where \( R = [1, e] \times [1, 4] \) is equal to
   (a) \( \frac{2}{3} \ln 4(e^{3/2} - 1) \)
   (b) 2
   (c) \( \frac{14}{3} \)
   (d) \( \frac{2}{3}(e^{-2} - 1) \)

2. Suppose that the integral \( \int \int_{D} f(x, y) \ dx \ dy \) is equal to \( \int_{0}^{\pi} \int_{2}^{3} r \ dz \ dr \). Find \( f(x, y) \):
   (a) \( f(x, y) = (x^2 + y^2)^{-1/2} \)
   (b) \( f(x, y) = 1 \)
   (c) \( f(x, y) = x^2 + y^2 \)
   (d) \( f(x, y) = (x^2 + y^2)^{-1} \)

3. Which of the following integrals is NOT equal to the volume under the paraboloid \( z = 5-x^2-y^2 \) and above the \( xy \) plane?
   (a) \( \int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5-x^2}}^{\sqrt{5-x^2}} 5 - x^2 - y^2 \ dy \ dx \)
   (b) \( \int_{0}^{2\pi} \int_{0}^{\sqrt{5}} \int_{0}^{5-r^2} r \ dz \ dr \ d\theta \)
   (c) \( \int_{0}^{2\pi} \int_{0}^{\sqrt{5}} 5 - r^2 \ dr \ d\theta \)
   (d) \( \int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5-y^2}}^{\sqrt{5-y^2}} \int_{0}^{5-x^2-y^2} dz \ dx \ dy \)

4. Using the Fundamental Theorem of Line Integrals, calculate the work done by the force field \( \mathbf{F} = (y + 1, x) \) on a particle moving on a path \( \mathbf{c}(t) = \left( \frac{1}{(\ln t)^{3/2}}, (\ln t) + 3 \right) \) as \( t \) goes from \( e \) to \( e^4 \).
   (a) \(-4\frac{7}{8} \)
   (b) integral cannot be evaluated without tables or computer
   (c) -4
   (d) \( 2\frac{7}{8} \)
5. Given the point \((-5, 0, 2)\) in Cartesian (rectangular) coordinates, its cylindrical coordinates are
   (a) \((5, \frac{\pi}{2}, 2)\)
   (b) \((2, \frac{3\pi}{2}, 5)\)
   (c) \((5, \pi, 2)\)
   (d) \((-5, \frac{3\pi}{2}, 2)\)

6. Classify the following three statements as True (T) or False (F) in the order (1), (2), (3).
   (1) If \(C\) is a circle, and \(\mathbf{F}\) is any vector field, then \(\int_C \mathbf{F} \cdot d\mathbf{s} = 0\) is always true.
   (2) If \(\int_C \mathbf{F} \cdot d\mathbf{s} < 0\), then the force field \(\mathbf{F}\) is hindering the progress of a particle on path \(C\).
   (3) If \(\mathbf{c}(t)\) is a flow line of \(\mathbf{F}\), then \(\int_a^b \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t) \, dt = 0\)
   (a) FFF
   (b) TTF
   (c) TFT
   (d) FTF

7. Given the point \((\sqrt{2}, 0, \sqrt{2})\) in Cartesian (rectangular) coordinates, its spherical coordinates are
   (a) \((\frac{1}{2}, 0, \pi)\)
   (b) \((2, \frac{\pi}{2}, \frac{\pi}{2})\)
   (c) \((\frac{1}{2}, \frac{\pi}{4}, \pi)\)
   (d) \((2, 0, \frac{\pi}{4})\)

8. The following integral represents the line integral along the geometric curve \(y = -x^2 + 4x\) from point \((4, 0)\) to point \((2, 4)\) of vector field \(\mathbf{F}\) (note, you are looking specifically at the parametrization of the curve):
   (a) \(\int_0^2 \mathbf{F}(-t, -t^2 - 4t) \cdot (-1, -2t - 4) \, dt\)
   (b) \(\int_0^2 \mathbf{F}(4 - t, -(4 - t)^2 + 4(4 - t)) \cdot (-1, 4 - 2t) \, dt\)
   (c) \(\int_2^4 \mathbf{F}(t, -t^2 + 4t) \cdot (1, -2t + 4) \, dt\)
   (d) None of the above integrals represent the described line integral.
9. Match the integrals with the volume that they represent.

(1) \[\int_0^{2\pi} \int_0^\pi \int_0^1 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta\]

(2) \[\int_0^{2\pi} \int_0^4 \int_0^6 r \, dr \, d\theta\]

(3) \[\int_0^2 \int_{-(y-8)}^{-(y-2)} \int_3^0 \, dx \, dy\]

(4) \[\int_0^{2\pi} \int_0^5 \int_0^5 \int_r^5 \, z \, dz \, dr \, d\theta\]

(i) volume of a parallelogram box (ii) volume of a sphere
(iii) volume of a cone (iv) volume of a cylinder

(a) 1 - ii, 2 - iv, 3 - i, 4 - iii
(b) 1 - iv, 2 - iii, 3 - ii, 4 - i
(c) 1 - iii, 2 - i, 3 - i, 4 - iv
(d) 1 - ii, 2 - iii, 3 - iv, 4 - i

10. Consider the integral \[\int_1^2 \int_{4x^2}^{16} f(x, y) \, dy \, dx\]. Changing the order of integration makes it equal to:

(a) \[\int_0^{\sqrt{4/\pi}} \int_4^{\sqrt{\pi}} f(x, y) \, dx \, dy\]

(b) \[\int_4^{\sqrt{\pi}} \int_0^{\sqrt{4/\pi}} f(x, y) \, dx \, dy\]

(c) \[\int_1^{\sqrt{4/\pi}} \int_0^{\sqrt{\pi}} f(x, y) \, dx \, dy\]

(d) \[\int_4^{\sqrt{4/\pi}} \int_1^{\sqrt{\pi}} f(x, y) \, dx \, dy\]
Part II: You can earn partial credit on the next four problems.

11. (12 points) Set up the following integral using the most computationally convenient coordinates (you do not have to solve): \[ \iiint_{W} \sqrt{x^2 + y^2 + z^2} \, dV \] if \( W \) is the portion of the sphere centered at \((0, 0, 0)\) with radius 4 that is bounded by the planes \( x \geq 0, y \geq 0, \) and \( z \geq 0.\)

12. (12 points) Evaluate the integral: \[ \int_{0}^{\ln 10} \int_{e^x}^{10} \frac{1}{\ln y} \, dy \, dx \]

13. (13 points) Set up the integral that represents the volume of the region that lies inside of both the cone \( z = 10 - \sqrt{x^2 + y^2} \) and the cylinder \( x^2 + y^2 = 4, \) and is bounded by the \( xy \) plane. Use the most computationally convenient coordinates; you do not have to solve.

14. (13 points) Find the work done by a force field \( \mathbf{F} = y^2 \mathbf{i} + (y - x) \mathbf{j} \) on a particle moving around the triangle in the plane given by the line \( y = \frac{x}{4}, \) and the points (in order of movement) \((0, 0), (4, 1), \) and \((4, 0).\)