Part I: Multiple choice. Each problem is worth 5 points.

1. The integral $\int \int_R \frac{\sqrt{y}}{x} \, dx \, dy$, where $R = [1, e] \times [1, 4]$ is equal to
   (a) $\frac{2}{3} \ln 4(e^{3/2} - 1)$
   (b) 2
   (c) $\frac{14}{3}$
   (d) $\frac{2}{3}(e^{-2} - 1)$

2. Suppose that the integral $\int \int_D f(x, y) \, dx \, dy$ is equal to $\int_0^\pi \int_2^3 dr \, d\theta$.
   Find $f(x, y)$:
   (a) $f(x, y) = (x^2 + y^2)^{-1/2}$
   (b) $f(x, y) = 1$
   (c) $f(x, y) = x^2 + y^2$
   (d) $f(x, y) = (x^2 + y^2)^{-1}$
3. Which of the following integrals is NOT equal to the volume under the paraboloid \( z = 5 - x^2 - y^2 \) and above the \( xy \) plane?

(a) \( \int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5-x^2}}^{\sqrt{5-x^2}} (5 - x^2 - y^2) \, dy \, dx \)

(b) \( \int_{0}^{2\pi} \int_{0}^{\sqrt{5}} \int_{0}^{\sqrt{5-r^2}} r \, dz \, dr \, d\theta \)

(c) \( \int_{0}^{2\pi} \int_{0}^{\sqrt{5}} (5 - r^2) \, dr \, d\theta \)

(d) \( \int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5-y^2}}^{\sqrt{5-y^2}} (5 - x^2 - y^2) \, dz \, dx \, dy \)

4. Using the Fundamental Theorem of Line Integrals, calculate the work done by the force field \( \mathbf{F} = (y + 1, x) \) on a particle moving on a path \( \mathbf{c}(t) = \left( \frac{\ln(t) + 3}{t^{3/2}}, \ln(t) + 3 \right) \) as \( t \) goes from \( e \) to \( e^4 \).

(a) \(-4\frac{7}{8}\)

(b) integral cannot be evaluated without tables or computer

(c) \(-4\)

(d) \(2\frac{7}{8}\)
5. Given the point \((-5, 0, 2)\) in Cartesian (rectangular) coordinates, its cylindrical coordinates are
   (a) \((5, \frac{\pi}{2}, 2)\)
   (b) \((2, \frac{3\pi}{2}, 5)\)
   (c) \((5, \pi, 2)\)
   (d) \((-5, \frac{3\pi}{2}, 2)\)

6. Classify the following three statements as True (T) or False (F) in the order (1), (2), (3).
   (1) If \(C\) is a circle, and \(F\) is any vector field, then \(\int_C F \cdot ds = 0\) is always true.
   (2) If \(\int_C F \cdot ds < 0\), then the force field \(F\) is hindering the progress of a particle on path \(C\).
   (3) If \(c(t)\) is a flow line of \(F\), then \(\int_a^b F(c(t)) \cdot c'(t) \, dt = 0\)
   (a) FFF
   (b) TTF
   (c) TFT
   (d) FTF
7. Given the point \((\sqrt{2}, 0, \sqrt{2})\) in Cartesian (rectangular) coordinates, its spherical coordinates are

(a) \((\frac{1}{2}, 0, \pi)\)
(b) \((2, \frac{\pi}{2}, \frac{\pi}{2})\)
(c) \((\frac{1}{2}, \frac{\pi}{4}, \pi)\)
(d) \((2, 0, \frac{\pi}{4})\)

8. The following integral represents the line integral along the geometric curve \(y = -x^2 + 4x\) from point \((4, 0)\) to point \((2, 4)\) of vector field \(\mathbf{F}\) (note, you are looking specifically at the parametrization of the curve):

(a) \(\int_{0}^{2} \mathbf{F}(-t, -t^2 - 4t) \cdot (-1, -2t - 4) \, dt\)
(b) \(\int_{0}^{2} \mathbf{F}(4 - t, -(4 - t)^2 + 4(4 - t)) \cdot (-1, 4 - 2t) \, dt\)
(c) \(\int_{2}^{4} \mathbf{F}(t, -t^2 + 4t) \cdot (1, -2t + 4) \, dt\)
(d) None of the above integrals represent the described line integral.
9. Match the integrals with the volume that they represent.

(1) \[ \int_0^{2\pi} \int_0^\pi \int_0^1 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \]

(2) \[ \int_0^{2\pi} \int_0^4 \int_0^6 r \, dr \, d\theta \]

(3) \[ \int_0^2 \int_{-(y-8)}^{-(y-2)} 3 \, dx \, dy \]

(4) \[ \int_0^{2\pi} \int_0^5 \int_r^5 r \, dz \, dr \, d\theta \]

(i) volume of a parallelogram box
(ii) volume of a sphere
(iii) volume of a cone
(iv) volume of a cylinder

(a) 1 – ii, 2 – iv, 3 – i, 4 – iii
(b) 1 – iv, 2 – iii, 3 – ii, 4 – i
(c) 1 – iii, 2 – i, 3 – i, 4 – iv
(d) 1 – ii, 2 – iii, 3 – iv, 4 – i

10. Consider the integral \[ \int_2^1 \int_{16}^4 f(x, y) \, dy \, dx \]. Changing the order of integration makes it equal to:

(a) \[ \int_0^4 \int_{\sqrt{y}}^{2} f(x, y) \, dx \, dy \]

(b) \[ \int_4^{16} \int_{\sqrt{y}}^{\frac{y^2}{4}} f(x, y) \, dx \, dy \]

(c) \[ \int_4^{16} \int_{\sqrt{y}}^{\frac{y^2}{4}} f(x, y) \, dx \, dy \]

(d) \[ \int_4^{4y^2} \int_1^1 f(x, y) \, dx \, dy \]
Part II: You can earn partial credit on the next four problems.

11. (12 points) Set up the following integral using the most computationally convenient coordinates (you do not have to solve): \[ \int \int \int_{W} \sqrt{x^2 + y^2 + z^2} \, dV \]
if \( W \) is the portion of the sphere centered at \((0, 0, 0)\) with radius 4 that is bounded by the planes \( x \geq 0, y \geq 0, \) and \( z \geq 0. \)
12. (12 points) Evaluate the integral: \[ \int_{e^2}^{10} \int_{\ln y}^{10} \frac{1}{\ln y} \, dy \, dx \]
13. (13 points) Set up the integral that represents the volume of the region that lies inside of both the cone \( z = 10 - \sqrt{x^2 + y^2} \) and the cylinder \( x^2 + y^2 = 4 \), and is bounded by the \( xy \) plane. Use the most computationally convenient coordinates; you do not have to solve.
14. (13 points) Find the work done by a force field \( \mathbf{F} = y^2 \mathbf{i} + (y - x) \mathbf{j} \) on a particle moving around the triangle in the plane given by the line \( y = \frac{x}{4} \), and the points (in order of movement) \((0, 0), (4, 1), \) and \((4, 0)\).