A Let \( \vec{F}(x, y) = \left( e^{y^2}, \frac{2xy}{x^2 y} \right) \) and \( C \) be the curve that runs in a straight line from \((0, 0)\) to \((0, 2)\) and then clockwise along a circular arc to \((2, 0)\).

Find the flux of \( \vec{F} \) across \( C \).

Hint: computing directly is very, very difficult - try to find a way that involves computing with easier integrals.

B Is the vector field \( \vec{F} = \left( \frac{xy^2 \sqrt{1000 + xy}}{x^2 y \sqrt{1000 + xy}}, \frac{x^2 \sqrt{1000 + xy} + 3}{2 \sqrt{1 + (x^2 + y^2)^{3/2}}} \right) \) conservative on the disc of radius 5 around the origin? Justify your answer.

Find \( \int_C \vec{F} \cdot d\vec{r} \) where \( C \) is the portion of the ellipse \( x^2 + 4y^2 = 1 \) with \( x \geq 0 \) and \( y \geq 0 \), oriented counter-clockwise.

C If \( r \) is the radial function \( r = \sqrt{x^2 + y^2} \) show that

\[
\text{curl} \left( \begin{pmatrix} -yf(r) \\ xf(r) \end{pmatrix} \right) \cdot \hat{k} = 2f(r) + rf'(r).
\]

Use this to evaluate

\[
\iint_D 2\sqrt{1+(x^2+y^2)^{3/2}} + \frac{3}{2} \frac{(x^2+y^2)^{3/2}}{\sqrt{1+(x^2+y^2)^{3/2}}} \, dA
\]

where \( D \) is the unit circle centered at the origin.