Answer ALL questions. Unless instructed otherwise, you should show ALL your work and simplify your final answer as much as possible. Please box your final answer to each part.

**Problem 1:** [8 pts] A metal sheet is bent into the shape described by rotating the curve $z = x^2$ with $0 \leq x \leq \sqrt{2}$ all the way around the $z$-axis. The metal has density $\rho(x, y, z) = 3 \text{ kg}/m^2$. Find the total mass of the sheet.
**Problem 2:** [8 pts] Find the flux of the vector field $\vec{F} = \left( \begin{array}{c} \frac{z^2 x}{3} \\ \frac{1}{3} y^3 + \tan z \\ \frac{x^2 z + y^2}{x^2 + y^2} \end{array} \right)$ across the surface $x^2 + y^2 + z^2 = 1$, $z \geq 0$ oriented upwards.
**Problem 3:** [9 pts] An infinite cylinder filling the region $x^2 + z^2 \leq 9$ has charge density $q(x, y, z) = \sqrt{x^2 + z^2}$. Find the electric field generated by this cylinder.

Some useful identities in cylindrical coordinates $(r, \theta, y)$

\[
\nabla f = \frac{\partial f}{\partial r} \hat{e}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{e}_\theta + \frac{\partial f}{\partial y} \hat{j}
\]

\[
\text{div } \vec{F} = \frac{1}{r} \frac{\partial}{\partial r} \left( r F_r \right) + \frac{1}{r} \frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_y}{\partial y}
\]

\[
\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial y^2}
\]