The Invertible Matrix Theorem

Let $A$ be a square $n \times n$ matrix. Then the following statements are equivalent.

a. $A$ is an invertible matrix
b. $A$ is row equivalent to the $n \times n$ identity matrix
c. $A$ has $n$ pivot positions
d. The equation $Ax = 0$ has only the trivial solution
e. The columns of $A$ form a linearly independent set
f. The linear transformation $x \mapsto Ax$ is one-to-one
g. The equation $Ax = b$ has at least one solution for each $b \in \mathbb{R}^n$
h. The columns of $A$ span $\mathbb{R}^n$
i. The linear transformation $x \mapsto Ax$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$
j. There is an $n \times n$ matrix $C$ such that $CA = I$
k. There is an $n \times n$ matrix $D$ such that $AD = I$
l. $A^T$ is an invertible matrix