Additional Homework Problems
October 17, 2005

Exercise 1.

a. Let $A$ be an $n \times n$ matrix. Suppose that $A^k = 0$ for some integer $k \geq 1$. Show that $I - A$ is invertible and that

$$(I - A)^{-1} = I + A + A^2 + \cdots + A^{k-1}.$$  

b. Let

$$C = \begin{pmatrix} 8/5 & 1/5 & -6/5 \\
-1/5 & 8/5 & 2/5 \\
3/5 & -4/5 & -1/5 \end{pmatrix}.$$  

Use part (a) to show that $C$ is invertible and find $C^{-1}$ without row reduction. [Hint: Write $C = I - A$ and show that $A^3 = 0$. Then apply part (a).]

Exercise 2. Let $V = \mathbb{R}^+$ be the set of positive real numbers. We define addition in $V$ as follows: if $x$ and $y$ are in $V$ then

$$x \oplus y = xy$$

where the right-hand side is ordinary multiplication of real numbers. If $c$ is a scalar (real number) and $x$ is in $V$ then we define scalar multiplication by

$$c \odot x = x^c$$

where the right-hand side is ordinary exponentiation of a real number. Show, by verifying the 10 axioms, that $V$ together with the operations $\oplus$ and $\odot$ is a vector space.

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1Such matrices are called nilpotent.