1. Suppose that $V$ and $W$ are vector spaces and that $T : V \to W$ is a linear transformation. If $\{ \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p \}$ are vectors in $V$ and if $\{ T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_p) \}$ is linearly independent, then show that $\{ \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p \}$ is linearly independent.

2. Suppose that $V$ and $W$ are vector spaces and that $T : V \to W$ is a linear transformation. Suppose that $\{ \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p \}$ is a linearly independent set of vectors in $V$. Must it be the case that $\{ T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_p) \}$ is linearly independent?

3. Suppose that $V$ and $W$ are vector spaces and that $T : V \to W$ is a one-to-one linear transformation. Suppose that $\{ \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p \}$ is a linearly independent set of vectors in $V$. Must it be the case that $\{ T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_p) \}$ is linearly independent?

Let $V$ and $W$ be vector spaces. A linear transformation $T : V \to W$ which is both one-to-one and onto is called an isomorphism of $V$ onto $W$. An isomorphism $T$ is invertible, and we proved its inverse, $T^{-1} : W \to V$, is also a linear map. Note that $T^{-1}$ is also one-to-one and onto.

4. Suppose that $T : V \to W$ is an isomorphism of $V$ onto $W$.
   
   (a) Show that $H$ is a subspace of $V$ if and only if $T(H) := \{ T(\mathbf{v}) \in W : \mathbf{v} \in H \}$ is a subspace of $W$.

   (b) Let $H$ be a subspace of $V$. Show that $\{ \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p \}$ is a basis for $H$ if and only if $\{ T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_p) \}$ is a basis for $T(H)$. 