Math 22 HW 6

32. (4.5 points) ATA has \( m \times n \) pivots; AAT has \( n \times n \) pivots (\( m \geq n \)).

So ATA is not invertible, but AAT is. But \( \det(AA^T) \approx 10^{-16} \) (i.e., close to zero, within rounding error).

4.2. (3.5 points) (a) \( A = \begin{bmatrix} 1 & 0 & 1/3 & 0 & -1/3 \\ 0 & 1 & 0 & 2 \sqrt{2}/3 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \) col \( 1,2,4 \) are pivot columns of \( A \).

Thus \( \text{Null}(A) \) only spans the column space.

(b) \( \text{Null}(A) = \begin{pmatrix} -1/3 \\ 1/3 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} -10/3 \\ 2 \sqrt{2}/3 \\ 0 \\ 4 \\ 1 \end{pmatrix} x_5 \)

(c) One-to-one \( \Rightarrow \text{Null}(A) = 0 \); onto \( \Rightarrow \text{Col}(A) = \mathbb{R}^4 \).

4.3. (3 points) The vectors are linearly dependent and do not span \( \mathbb{R}^2 \).

19. (2 points) Basis for Null \( A \) : \( \begin{bmatrix} -1/2 \\ 1/2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \), \( \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \).

Basis for Col \( A \) : \( \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \), \( \begin{bmatrix} 4 \\ 0 \\ 0 \\ 1 \end{bmatrix} \).

22. (3 points) a. False. See def. of a basis.

b. True. By the spanning set theorem.

C. True. See the subsection "Two Views of a Basis."

d. False. See two paragraphs before Example 8.

e. False. See the warning after theorem 6.

24. (4 points) Let \( A = [v_1, \ldots, v_n] \). Since \( A \) is square and its columns are linearly independent, its columns also span \( \mathbb{R}^n \) by the Invertible Matrix Theorem.

So \( [v_1, \ldots, v_n] \) is a basis for \( \mathbb{R}^n \).
4.4 (3 points) \[ \begin{bmatrix} 5 \\ -7 \end{bmatrix} \]

30 (3 points) Linearly dependent since the coordinate vectors are linearly dependent.

4.5 (2 points) \[ \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{dim is 3.} \]

22 (3 points) No Lagrange polynomial is a linear combination of the Lagrange polynomials of lower degree.

By Theorem 4.1 (4.3), the set of polynomials is linearly independent.

Since this set contains four vectors and \( \mathbb{P}_3 \) is four-dimensional, the set is a basis of \( \mathbb{P}_3 \) by the Basis Theorem.

4.6 (2 points) \( \dim \text{null} (A) = 2 \).

It is impossible for col A to be \( \mathbb{R}^4 \) since the vectors in col A have 5 entries.

col A is a four-dimensional subspace of \( \mathbb{R}^5 \).