5.3 (2 points) \( \begin{bmatrix} 1 & 15 \, 90 \end{bmatrix} \)

10 (2 points) \( P = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix} \)

12 (2 points) \( P = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \).

26 (2 points) Yes, if the third eigen-space is only one-dimensional. In this case, the sum of the dimensions of the eigenspaces will be six, whereas the matrix is 7x7.

See Theorem 7(b).

28 (2 points) If \( A \) has \( n \) linearly independent eigenvectors, then by the Diagonalization Theorem,

\[ A = PDP^{-1} \text{ for some invertible } P \text{ and diagonal } D. \]

Then \( A^T = (PDP^{-1})^T = (P^{-1})^TDP^T = (P^T)^{-1}DP^T = OD_0D^{-1}, \)

where \( D = (P^T)^{-1}. \)

Thus \( A \) is diagonalizable.

By the Diagonalization Theorem, the columns of \( D \) are \( n \) linearly independent eigenvectors of \( A^T. \)

4. 9 (12 points) Each food will be preferred equally, because \( \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} \) is the steady-state vector.

17 (3 points) a. The entries in a column of \( P \) sum to 1. A column in the matrix \( P^{-1} \) has the same entries as in \( P \) except that one of the entries is decreased by 1.

Hence each column sum is 0.

b. By (a), the bottom row of \( P^{-1} \) is the negative of the sum of other rows.
C. By (b) and the Gram-Schmidt Theorem, the bottom row of $P: \tilde{A}$ can be removed and the remaining
(x-1) rows will still span the row space.

1. By the Rank Theorem and (c), the dimension of the column space of $P: \tilde{A}$ is less than or
Hence the null space is nontrivial.

using $\text{null}(P = \text{eye}(3), r_r)$

then dividing by the sum of the column vec.

and $b$ have the highest rank. (jointly highest)

2. It will increase, approaching 1, so $b$ can beat the system, hereby.

3. $P = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, steady state $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, ie $c$ gets all the ranking.

This is not fair!

4. $A_{\text{steady state}} = \begin{bmatrix} 0.111 & 1/3 \\ 0.111 & 1/3 \\ 0.111 & 1/3 \end{bmatrix}$

5. (Bonus): if $x = dPx + (1-d)\bar{e}$ the column sums are $S = dS + (1-d)\bar{e}$.

$S = \text{sum}(x)$, unknown.

\[ S = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \]

5.6 (12 points) Saddle point. Eigenvectors: 1.1, 0.8. Convergent region: line through $(0,0)$ and $(0,1)$

6.1 (12 points) $\begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix}$

14 (points) $2\sqrt{17}$

Note an eigenvector issue - is it a lin. system or an eigenvector??

$(I-dP)x = (1-d)e$ is lin. system $Ax = b$.

But $\bar{e}$ can be written $M\bar{x}$ where $M = \begin{bmatrix} y_1 & y_2 & \cdots \\ y_2 & y_3 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$, since $\bar{x}$ is a prob. vector

Thus it is also an eigenvector problem!

Note the matrix $M$ corresponds to randomly jumping anywhere in the web.