6.1 (16 points) A. True. See the definition of U. V.
B. True. See Theorem 1(c).
C. True. See the discussion of Fig. 5.
D. False. Counterexample [6.6.]
E. True. See the box following Example 6.

6.2 (10 points) Show \( \mathbf{u}_1 \cdot \mathbf{u}_2 = 0, \mathbf{u}_2 \cdot \mathbf{u}_3 = 0, \mathbf{u}_3 \cdot \mathbf{u}_1 = 0 \).

Use Theorem 4 and observe that three linearly independent vectors in \( \mathbb{R}^3 \) form a basis.

\[ \mathbf{x} = \frac{1}{2} \mathbf{u}_1 + \frac{1}{2} \mathbf{u}_2 + \frac{1}{2} \mathbf{u}_3. \]

Use projection formula

\[ \mathbf{x} = \mathbf{u}_1 - \frac{1}{2} \mathbf{u}_1; \quad i = 1,2,3. \]

14 (2 points) \( y = \begin{bmatrix} 14/5 \\ 2/15 \end{bmatrix} + \begin{bmatrix} -4/15 \\ 28/15 \end{bmatrix} \)

20 (5 points) Not orthogonal. Orthogonal set \( \begin{bmatrix} -2/15 \\ 2/15 \end{bmatrix} \), \( \begin{bmatrix} 1/5 \\ 0 \end{bmatrix} \)

26 (2 points) (Orthogonal) \( U^T U = I \).

If \( \mathbf{V} = \mathbf{U}^T \) then \( \mathbf{U}^T \mathbf{U} = I \) (In particular, the columns of \( \mathbf{U}^T \) are linearly independent and hence form a basis for \( \mathbb{R}^n \)).

6.3 (2 points) \( U = 2 \mathbf{u}_1 + \frac{2}{3} \mathbf{u}_2 + \frac{12}{7} \mathbf{u}_3 - \frac{2}{7} \mathbf{u}_4 \)

8 (5 points) \( y = \begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix} + \begin{bmatrix} -5/2 \\ 1/2 \end{bmatrix} \)

24 (6 points) A. By hypothesis, the vectors \( \mathbf{u}_1, \ldots, \mathbf{u}_p \) are pairwise orthogonal, and the vectors \( \mathbf{u}_{p+1}, \ldots, \mathbf{u}_n \)

are pairwise orthogonal.

Also, \( \mathbf{u}_i \cdot \mathbf{u}_j = 0 \) for any \( i \) and \( j \) because the \( 
\)s are in the orthogonal complement of \( \mathbf{u}_i \).
b. By QR, write \( y = Qz + x \) as in the Orthogonal Decomposition Theorem, with \( y \in \mathbb{R}^n \).

Then there exist scalars \( a_1, \ldots, a_p \) and \( d_1, \ldots, d_q \) such that

\[
y = Qz + x = a_1 v_1 + \cdots + a_p v_p + d_1 u_1 + \cdots + d_q u_q.
\]

for any \( y \in \mathbb{R}^n \).

Thus \( \{v_1, \ldots, v_p, u_1, \ldots, u_q\} \) spans \( \mathbb{R}^n \).

C. The set \( \{v_1, \ldots, v_p, u_1, \ldots, u_q\} \) is linearly independent by (a), span \( \mathbb{R}^n \) by (b), and thus is a basis for \( \mathbb{R}^n \). Hence \( \dim V + \dim W^\perp = n + q = \dim \mathbb{R}^n = n \).

6.5 (3 points) a. \[
\begin{bmatrix}
12 & 8 \\
9 & 10
\end{bmatrix}
\begin{bmatrix}
12 \\
25
\end{bmatrix}
= \begin{bmatrix}
24 \\
15
\end{bmatrix}
\]
b. \( \lambda = \begin{bmatrix}
-3
\end{bmatrix} \).

20 (3 points) Suppose that \( A \neq 0 \). Then \( A^T A = A^T 0 = 0 \).

Since \( A^T A \) is invertible by hypothesis, \( x = 2 \).

\[ \therefore \text{ The columns of } A \text{ are linearly independent.} \]

7.1 (3 points) Orthogonal. \( U^T = U = \begin{bmatrix}
\sqrt{3}/2 & \sqrt{3}/2 \\
-1/2 & 1/2
\end{bmatrix} \) Cardioid clockwise rotation by \( 45^\circ \).

20 (2 points) \( P = \begin{bmatrix}
\sqrt{3}/2 & 1/3 & 2/3 \\
1/3 & 2/3 & 1/3 \\
2/3 & 1/3 & -1/3
\end{bmatrix} \), \( D = \begin{bmatrix}
13 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix} \).

35 (3 points) a. Note that \( (u \cdot v)^T u = u^T u = (u^T X) u \) because \( u^T X \) is a scalar.

\[ \because u^T (X - B) = u^T X - u^T (u^T u) u^T = u^T X - u X^T = 0. \]

b. \( B_{ij} = u_i u_j = u_j u_i = B_{ji} \Rightarrow B \) is symmetric.

\[ B^2 = uu^T uu^T = uu^T uu^T = uu^T = B. \]

C. \( Bu = uu^T \Rightarrow uu^T = 0 \cdot 1 = 0 \cdot u \)

\[ \therefore u \text{ is an eigen vector of } B \text{ with an eigenvalue of } 0. \]