For each described operation, find standard matrix $A$, and whether $T$ is onto and one-to-one.

a) $T(x_1, x_2) = (3x_1, -2x_1 + x_2, -x_2)$

$T: \mathbb{R}^2 \to \mathbb{R}^3$ (What are $u_1, u_2$?)

What size?

b) $T$ is reflection about line $x_2 = x_1$

($T: \mathbb{R}^2 \to \mathbb{R}^2$)

A =

tonner?

one-to-one?

c) $T: \mathbb{R}^3 \to \mathbb{R}^2$

projects the point $(x_1, y_2)$ downward vertically onto the $(x,y)$ plane

(the shadow of a point under the midday sun).

A =
tonner?

one-to-one?
For each described operation, find standard matrix $A$, and whether $T$ is onto and one-to-one.

\( T(x, y, z) = (3x, -2x, x, -z) \)

$T: \mathbb{R}^3 \to \mathbb{R}^4$ (What are $v$, $w$, $z$?)

2 vector cannot span $\mathbb{R}^3$

since would need a pivot in each of 3 rows for this.

\( A = \begin{bmatrix} 3 & 0 \\ -2 & 1 \\ 0 & -1 \end{bmatrix} \)

onto? No, since $Ax = b$ not consistent for all $b$ in $\mathbb{R}^3$.

one-to-one? Yes, since when $Ax = b$ is consistent, it is unique ($b'$ is image of single $x$).

\( T: \mathbb{R}^2 \to \mathbb{R}^2 \)

is reflection about line $x_2 = x_1$.

\( A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \)

onto? Yes since pivot in every row.

one-to-one? Yes since there are no free vars in $Ax = b$.

\( T: \mathbb{R}^3 \to \mathbb{R}^3 \)

projects the point $(x_1, y_2, z_3)$ down vertically onto the $(x_1, y_2)$ plane.

(Shadow of a point under the wighly sun).

\( A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \) in $\mathbb{R}^3$.

onto? Yes, since pivot in every row.

one-to-one? No, since in $Ax = b$, $x_3$ is free var, not unique.