Math 22: Linear Algebra. PRACTISE
MIDTERM 2 — ANSWERS

August 4, 2006

1. a) +8, b) $\lambda = 1$ with $v = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$, and $\lambda = 6$ with $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

2. Col $A$ has basis $\begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 2 \\ -6 \end{bmatrix}$. Row $A$ has basis $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -2 \\ 5 \\ 6 \end{bmatrix}$.

Proof: basis for $H$ must be lin indep vectors, which also lie in $V$. However, no more than dim $V$ vectors can be lin indep in $V$. QED.

3. $L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -2 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

$LU$ is useful since once performed, $Ax = b$ can be solved for vectors $b$ with only $O(N^2)$ effort, where $N$ is typical size of matrix.

4. a) Nul $A$, so it’s a subspace, write out proof of Thm 2 (p. 227).

b) basis is the one vector $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$. dim $W = 1$.

5. (a) $\begin{bmatrix} -2 \\ -7 \\ 8 \end{bmatrix}$
(b) There are 2 basis vectors, so you know $[x]_B$ must have 2 components, call them $c_1$ and $c_2$.

$$c_1 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$$

must be satisfied, since this what the $B$-coords of $x$ mean. This is just a linear equation which we solve by row reduction of augmented matrix:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ -1 & 1 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{(R.E.F.)}$$

It is consistent (otherwise $x$ would not be in $H$). The unique solution is $[x]_B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

6. (a) False: $\mathbb{R}^2$ is not a subspace of $\mathbb{R}^3$ because its elements (2-component vectors) do not even come from $\mathbb{R}^3$ (the set of 3-component vectors). It is not even a subset.

(b) a linear transformation that is both one-to-one and onto

(c) False. $Ax$ is a unique object, so it cannot both be $\lambda_1 x$ and $\lambda_2 x$, which it would have to be if an eigenvector for both eigenvalues.

(d) Write the 3 polynomials in the standard basis, to get

$$\left\{ \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\} \quad \text{stack as cols, reduce to get} \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

All 3 pivots, so we have 3 linearly-independent vectors in $\mathbb{R}^3$, so they form a basis. (You could also instead have said they span $\mathbb{R}^3$). $P_2$ is isomorphic to $\mathbb{R}^3$ so the original polynomials also form a basis for $P_2$. 

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