Homework 1

using the limit definition to compute an integral

Instructions: You are encouraged to work out solutions to these problems in groups! Discuss problems with your classmates, the tutors and/or the instructors. After doing so, please write up your solutions legibly on a separate sheet (or sheets) of paper, showing all of your work (this part should be done on your own). When you are asked to give explanations, be sure to use complete sentences. You are welcome (and sometimes encouraged) to use calculators or computing devices.

In this problem set we will use the limit definition to compute the integral.

\[ \int_{0}^{8} (x^2 - 10x + 24) \, dx \]

Figure 1: The graph of \( f(x) = x^2 - 10x + 24 \)

You may use the following formulas without proof.

\[ \sum_{i=1}^{n} i = \frac{n(n + 1)}{2} \quad \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6} \]
1. (2 Points) Use the midpoint rule with \( n = 4 \) to estimate the integral.

2. (2 Points) Write a limit expression for the integral using right endpoints as sample points.

3. (2 Points) Find what the \( x_i \) are in this case and compute \( f(x_i) \) for \( f(x) = x^2 - 10x + 24 \).

4. (3 Points) Simplify and then compute the limit to find the integral.

5. (1 Points) Explain what area the integral represents.