1. Short answer problems.

(a) A certain mushroom changes shape as it grows, so its weight $W$ (measured in grams) is a complicated function of its height $h$ (measured in centimeters.) If you know that

$$\int_{3}^{5} \frac{dW}{dh} \, dh = 2,$$

then what can you conclude?

i. An average mushroom between 3 and 5 centimeters tall weights 2 grams.

ii. A 5 centimeter tall mushroom weighs 2 grams more than a 3 centimeter tall mushroom.

iii. A 5 centimeter tall mushroom weighs twice as much as a 3 centimeter tall mushroom.

iv. A mushroom between 3 and 5 centimeters in height that grows a small amount $\Delta h$ centimeters taller will gain about $2\Delta h$ grams in weight.

(b) $F(x) = \int_{-x}^{x} \cos^2(t) \, dt$.

What is the derivative $F'(x)$?

(c) TRUE or FALSE?

i. On any interval on which the function $f$ is differentiable,

$$\int f'(x) \, dx = f(x) + C.$$

ii. The antiderivative of the velocity function of a moving particle gives you the particle’s acceleration.

iii. If $f(x) = f(-x)$ for all $x$, and $f$ is continuous everywhere, then for all $a$,

$$\int_{-a}^{a} f(x) \, dx = 0.$$
2. (a) The sum
\[ 1^2 \Delta x + 3^2 \Delta x + 5^2 \Delta x + 7^2 \Delta x + 9^2 \Delta x \]
is the fifth left-endpoint Riemann sum \((L_5)\) approximating
\[ \int_a^b x^2 \, dx. \]
Provide the following numbers:
\[ a = \]
\[ b = \]
\[ \Delta x = \]

(b) If \( f \) is continuous and
\[ \int_0^{\sqrt{3}} f(x) \, dx = -1, \]
\[ \int_0^{3} f(x) \, dx = 1, \]
\[ \int_0^{9} f(x) \, dx = 4, \]
find
\[ \int_0^{3} xf(x^2) \, dx. \]
(You may not need to use all the given information.)

(c) What is
\[ \int \frac{1}{\cos^2(x)} \, dx? \]
3. (a) Evaluate 
\[ \int_0^\pi \sin^3 x \, dx. \]
(b) Evaluate 
\[ \int_{-1}^1 (x + 2)\sqrt{1 - x^2} \, dx \]
by writing it as a sum of integrals and interpreting one of them as an area.

4. A solid object has a base that is a right triangle. One leg has length 2'' and the other has length 4''. When it is sliced vertically with slices parallel to the 2'' leg of the base, the cross-sections are half-discs. Find its volume.

5. A covered aquarium has a rectangular base, and must be twice as tall as the width of the base. If material for the top of the aquarium costs $5 per square foot, material for the bottom costs $8 per square foot, and glass for the sides costs $2 per square foot, find the volume of the largest aquarium that can be built for $100 or less.