For the multiple choice questions, we omit the choices and just calculate the answer.

1. For the function
\[ f(x) = \frac{x - 1}{x^2} = x^{-1} - x^{-2}, \]
what is its domain?

**Solution.** This function is defined everywhere except \( x = 0 \); therefore, the domain is \((-\infty, 0) \cup (0, \infty)\), which is choice **B**.

2. For the same function as question 1, what are its vertical and horizontal asymptotes?

**Solution.** The line \( x = 0 \) is evidently a vertical asymptote, because \( f(x) \) is not defined there, and \( \lim_{x \to 0} f(x) = -\infty \). For horizontal asymptotes, we analyze \( \lim_{x \to \infty} f(x) \) and \( \lim_{x \to -\infty} f(x) \), which are both equal to 0. Therefore \( y = 0 \) is a horizontal asymptote, and so choice **A** is correct.

3. A weight that is suspended by a spring of length 1 meter bobs up and down with its height above the floor at time \( t \) seconds given by the formula
\[ h(t) = 2 + \frac{1}{\pi} \sin(\pi t). \]

Note that the function \( h(t) \) has the derivative
\[ h'(t) = \cos(\pi t). \]

At \( t = 1/2 \) seconds the object is

**Solution.** **C**, at its highest point. Indeed, \( h(t) \) is largest (and hence the object is highest) when \( \sin \pi t = 1 \), which occurs precisely when \( t = 1/2, 5/2, 9/2, \ldots \).

4. For the same situation as problem 3, but at time \( t = 1 \) second, the object is

**Solution.** **B**, moving its fastest downward. A graph of the function \( h \) will show that the object is moving downwards at time \( t = 1 \); also, the fact that \( h'(1) = \)
\( \cos(\pi) = -1 \) indicates that the object is moving downwards. As a matter of fact, the object is moving its fastest downwards, because \(-1\) is the most negative value that \( h'(t) \) takes.

5. The function \( f(x) = \cos x + \sin x + 1 \) is

\textit{Solution.} C, neither even nor odd. \( f(x) \) certainly is not odd, because \( f(0) = 2 \neq -f(0) \). Also, \( f(x) \) is not even, since \( f(\pi/2) = 2 \), while \( f(-\pi/2) = 0 \), which are not equal to each other.

6. Let \( f(x) = \ln(2x - 1) \) and \( g(x) = e^{2x} \). Then \( g(f(x)) \) is

\textit{Solution.} D, \( 4x^2 - 4x + 1 \). For readability, we will write \( \exp \) for \( e^n \). We have

\[ g(f(x)) = \exp(2 \ln(2x - 1)). \]

We can factor \( x^2 + 2x + 1 = (x + 1)^2 \), so

Since \( 2 \ln(2x - 1) = \ln(2x - 1)^2 \), we have

\[ g(f(x)) = \exp(\ln(2x - 1)^2) = (2x - 1)^2 = 4x^2 - 4x + 1. \]

7. Solve for \( x \), expressing your answer in terms of the natural logarithm function:

\[ 8^{x+2} = 5^{-x}. \]

\textit{Solution.} Take the natural log of both sides to obtain

\[ \ln 8^{x+2} = \ln 5^{-x} \iff (x + 2) \ln 8 = -x \ln 5 \]

Now we solve for \( x \) in the usual way:

\[ x(\ln 8 + \ln 5) = -2 \ln 8 \Rightarrow x = \frac{-2 \ln 8}{\ln 8 + \ln 5} \]

We can simplify the denominator to \( \ln 40 \), using \( \ln a + \ln b = \ln ab \). Therefore, the correct choice is C.

8. Find \( f'(x) \) when \( f(x) = 3x^2 - 5x + \sqrt{3} \).

\textit{Solution.} Use the power rule to find \( f'(x) = 6x - 5 \), which is choice A.

9. Consider the function \( f(x) = -x + 1 \), for \( x \leq 1 \), and \( x^2 \), for \( x > 1 \). Which of the following statements is true?

\textit{Solution.} C, that \( x = 1 \) is a discontinuity of \( f \) which is not removable, is correct. Indeed, \( \lim_{x \to 1^-} f(x) = 0 \), while \( \lim_{x \to 1^+} f(x) = 1 \). Therefore, no definition of \( f(x) \) at \( x = 1 \) will make \( f(x) \) continuous, since \( \lim_{x \to 1} f(x) \) does not even exist.
10. For the limit
\[ \lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4}, \]
which of the following is true?

Solution. D, the limit is 1/4, is correct. Notice that the numerator and denominator of the function are both 0 at \( x = 2 \), so we factor both of them and notice that the term \( x - 2 \) cancels:

\[
\frac{x^2 - 3x + 2}{x^2 - 4} = \frac{(x - 2)(x - 1)}{(x - 2)(x + 2)} = \frac{x - 1}{x + 2}
\]

where the rightmost equality holds whenever \( x \neq 2 \). Therefore, the limit of the original fraction as \( x \to 2 \) is \((2 - 1)/(2 + 2) = 1/4\).

11. Find the derivative of the function: \( f(x) = \sin(3x + 1) + \cos(2x - 3) \).

Solution. Use the chain rule, and the formulas for the derivative of \( \sin \) and \( \cos \), to obtain

\[ f'(x) = 3 \cos(3x + 1) - 2 \sin(2x - 3) \]

which is choice E, none of the above.

12. Find \( f'(x) \) if \( f(x) = \tan \frac{x + 1}{x - 1} \).

Solution. We use the chain rule to find

\[ f'(x) = \sec^2 \frac{x + 1}{x - 1} \cdot \left( \frac{x + 1}{x - 1} \right)' \]

The derivative of \( (x + 1)/(x - 1) \) can be evaluated using the quotient rule:

\[ \left( \frac{x + 1}{x - 1} \right)' = \frac{1 \cdot (x - 1) - 1 \cdot (x + 1)}{(x - 1)^2} = \frac{-2}{(x - 1)^2} \]

This gives choice C. We could have also calculated the derivative of \( (x + 1)/(x - 1) \) by rewriting it as follows:

\[ \frac{d}{dx} x + 1 = \frac{d}{dx} \left( 1 + \frac{2}{x - 1} \right) = \frac{-2}{(x - 1)^2} \]

13. The function \( f(x) = \sqrt{x} \) has the derivative \( f'(x) = 1/(2\sqrt{x}) \). Find the equation of the tangent line to the graph of \( y = f(x) \) at the point (4, 2).

Solution. The tangent line in question passes through the point \( (4, 2) \) and has slope equal to \( f'(4) \). Therefore, the slope is \( f'(4) = 1/4 \), and the equation of the tangent line can be found using the point-slope form for a line:
\[ y - 2 = 1/4(x - 4) \]

If we wanted to, we could write this in the form \( y = x/4 + 1 \).

14. Carefully work out the derivative of \( f(x) = 2x^2 - 3x \) using the formula

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

**Solution.** Plug in the expression for \( f(x) \) into the definition of \( f'(x) \):

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{2(x + h)^2 - 3(x + h) - (2x^2 - 3x)}{h}
\]

We expand out the terms in the numerator and simplify:

\[
2(x+h)^2-3(x+h)-(2x^2-3x) = 2(x^2+2hx+h^2)-3x-3h-2x^2+3x = 4hx+2h^2-3h
\]

Therefore, \( f'(x) \) is equal to

\[
f'(x) = \lim_{h \to 0} \frac{4hx + 2h^2 - 3h}{h} = \lim_{h \to 0} \frac{4x - 3 + 2h}{h} = 4x - 3.
\]

This is in agreement with what the power rule would give us.