1. (20) (Show all work). Find the solution to the boundary value problem:
\[ \frac{dy}{dx} - 3x^2y = x^2, \quad y(0) = 1/3. \]

2. (20) (Show all work). Find all points on the surface given by \( 3z = x^2 + xy \) where the normal line to the surface is parallel to the line given by: \( x = 2t + 1, \quad y = 4t - 1, \quad z = -3t + 7. \)

3. (20) (Show all work). The temperature at a point \((x, y, z)\) is given by the function 
\[ T(x, y, z) = 200e^{x^2-4y^2-9z^2}, \]
measured in degrees Celsius. Find the rate of change of the temperature at the point \((2, 1, 0)\) in the direction from the point \((2, 1, 0)\) toward the point \((3, 2, 5)\).

4. (20) (Show all work). Find the maximum and minimum values of \( f(x, y, z) = x - 2y + 5z \) on the sphere \( x^2 + y^2 + z^2 = 30. \)

5. (70) Multiple Choice Circle the correct response. (No partial credit will be given)

(a) \[ \frac{1}{3 + 4i} = \]

A. \( \frac{1}{3} + \frac{1}{4}i \)  
B. \( \frac{1}{3} - \frac{1}{4}i \)  
C. \( \frac{3}{25} - \frac{4}{25}i \)  
D. \( \frac{3}{5} - \frac{4}{5}i \)  
E. none of the above

(b) What is the arclength of the piece of the parabola \( y = x^2 \) from \((0,0)\) to \((2,4)\)?

A. \[ \int_0^4 (1 + 2t) \, dt \]  
B. \[ \int_0^4 \sqrt{1 + 4t^2} \, dt \]  
C. \[ \int_0^2 \sqrt{t^2 + t^4} \, dt \]  
D. \[ \int_0^2 \sqrt{1 + 4t^2} \, dt \]  
E. none of the above

(c) The radius of convergence of the series \[ \sum_{n=1}^{\infty} \frac{(x - 5)^{2n}}{n^3 3^n} \] is

A. 1  
B. 3  
C. 5  
D. 3/5  
E. none of the above
(d) It is well-known by Math 8 students that \( \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots \). If we truncate this series after some point and use the partial sum as an approximation for \( \pi/4 \), what is the least number of terms needed in the partial sum so that the error is less than .01?

A. 49  B. 50  C. 98  D. 99  E. none of the above

(e) \( \left( \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \right) \left( \sum_{n=0}^{\infty} \frac{(-1)^n(3x)^{2n+1}}{(2n+1)!} \right) \) is a solution to which differential equation?

A. \( y'' + 5y' + 6y = 0 \)  B. \( y'' - 5y' + 6y = 0 \)
C. \( y'' + 4y' + 13y = 0 \)  D. \( y'' - 4y' + 13y = 0 \)
E. none of the above

(f) Let \( f(x, y) = |\langle x, y \rangle| \). What is the largest set on which \( f \) is continuous?

A. all of \( \mathbb{R}^2 \)  B. all of \( \mathbb{R}^2 \) except the axes
C. all of \( \mathbb{R}^2 \) except the origin  D. all of \( \mathbb{R}^2 \) except the \( x \)-axis
E. none of the above

(g) If \( f(x, y) = \int_{y}^{x} \cos(t^3) \, dt \), then \( \frac{\partial f}{\partial y} = \)

A. \( \cos(y^3) \)  B. \( 3y^2 \cos(y^3) \)  C. \( \sin(y^3) \)
D. \( -3y^2 \sin(y^3) \)  E. none of the above

(h) Suppose that you are given a function \( f(x, y) \) and vectors \( \mathbf{u} = \langle \frac{1}{2}, -\frac{\sqrt{3}}{2} \rangle \) and \( \mathbf{v} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle \). If \( (D_u f)(x_0, y_0) = 2 \) and \( (D_v f)(x_0, y_0) = -1 \), then \( \frac{\partial f}{\partial x}(x_0, y_0) = \)
A. $\frac{1}{2}$  
B. 1  
C. 2  
D. $\sqrt{3}$  
E. none of the above

(i) Suppose that $f$ is a function of the variables $x$, $y$, and $z$, and that $x$ is a function of the variables $s$ and $t$, $y$ is a function only of $s$ and $z$ is a function only of the variable $t$. What is the correct expression for $\frac{\partial f}{\partial t}$?

A. $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$  
B. $\left( \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} \right) \frac{\partial f}{\partial t}$  
C. $\frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial t}$  
D. $\frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial t}$  
E. none of the above

(j) The altitude of a right circular cylinder is 8 inches and is increasing at a rate of .8 in/min. The base radius is 10 inches and is decreasing at a rate of .05 in/min. The volume ($V = \pi r^2 h$) is changing at the rate of

A. $72\pi$  
B. $-72\pi$  
C. $88\pi$  
D. $-88\pi$  
E. $800\pi$

(k) Let $f(x, y, z) = xy + yz^2 + xz^3$ and $v = (-2, 0, 1)$. What can be said about the change in $f$ at the point $(2, 0, 3)$ in the direction indicated by $v$?

A. strictly increasing  
B. strictly decreasing  
C. unchanging  
D. first increasing, then decreasing  
E. impossible to be determined

(l) Suppose that $f(x, y)$ has continuous second partial derivatives and a critical point at $(2, 3)$. Suppose further that $f_{xx}(2, 3) = -3$, $f_{xy}(2, 3) = 2$, and $f_{yy}(2, 3) = -4$. Classify the critical point.

A. local minimum  
B. local maximum  
C. saddle point  
D. local extremum, but can’t determine which  
E. cannot be determined
Suppose that there is a function $f(x, y)$ having continuous first partial derivatives with $f_x = (x^2 - 4)(y - 3)$ and $f_y = (y^2 - 9)(x - 2)$. How many critical points does $f$ have in $\mathbb{R}^2$?

A. 2  
B. 4  
C. 5  
D. 6  
E. infinitely many

Suppose that the graph of $z = f(x, y)$ represents the surface of a mountain, and you are standing at a point $(x_0, y_0, z_0)$ on the surface. You are told that the gradient of $f$ at $(x_0, y_0)$ is $\nabla f(x_0, y_0) = (1, 3)$. If you move in the direction of the gradient, what is your initial angle of elevation?

A. $\tan^{-1} 3$  
B. $\cos^{-1} 3$  
C. $\tan^{-1} \sqrt{10}$  
D. $\cos^{-1} \sqrt{10}$  
E. none of the above