LECTURE OUTLINE
Partial Derivatives

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Math 8
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Goals

Partial Derivatives

\[ \frac{\partial f}{\partial x}(x, y) \]

Partial Differential Equations

Tangent Planes
Review: Graph

Example: \( f(x, y) = \cos(xy)e^{\frac{-x^2-y^2}{10}} \) with domain 
\(-5 \leq x \leq 5 \) and \(-5 \leq y \leq 5 \).
Example: \( f(x, y) = \cos(xy)e^{\frac{-x^2-y^2}{10}} \) with domain 
\[-5 \leq x \leq 5 \text{ and } -5 \leq y \leq 5.\]
Partial Derivatives

\( \frac{\partial f}{\partial x}(x, y) \) means take the derivative in \( x \) viewing \( y \) as constant, in other words,

\[
\frac{\partial f}{\partial x}(x, y) = f_x(x, y) = \lim_{h \to 0} \frac{f(x + h, y) - f(x, y)}{h}.
\]

**Ex:** Find \( f_x \) and \( f_y \) when \( \frac{xy}{\sqrt{x^2 + y^2}} \).
Higher Derivatives

Let \( f(x, y) = x^3 - y^3 \). Find \( f_{xx} \) and \( f_{yy} \).

A partial differential equation (PDE) is an equation like this:

\[
f_{xx} + f_{yy} = 0
\]

This is called Laplace’s Equation. We try and find solutions to a PDE, namely functions that solve the given equation.

Find a solution to Laplace’s equation.
We can also view a variable as *indexing* a family of functions in the other variable (often time).

\[ f(x, t) = \frac{e^{-\frac{x^2}{4t}}}{\sqrt{4\pi t}} \]
Another PDE

\[ f(x, t) = \frac{e^{-\frac{x^2}{4t}}}{\sqrt{4\pi t}} \]

**Ex:** Confirm \( f_t = f_{xx} \), the heat equation.
**Tangent Planes**

Example: \( f(x, y) = x^2 - y^2 \) at \((0, 0, 0)\).
Example: \( f(x, y) = x^2 - y^2 \) at \((0, 0, 0)\). Zoom in towards \((0, 0, 0)\).
Example: \[ f(x, y) = x^2 - y^2 \] at \((0, 0, 0)\). Zoom in towards \((0, 0, 0)\) and we see a plane, the tangent plane.
In other words: near \( (x_0, y_0) \) we have that \( f(x, y) \) looks like

\[
z = f(x_0, y_0) + \nabla f \cdot (x - x_0, y - y_0).
\]

**Example:** \( f(x, y) = x^2 - y^2 \) near \( (0, 0, 0) \).
Example: \( f(x, y) = x^2 - y^2 \) near \((0, 0, 0)\).
A Cruel and UNUSUAL example

The tangent plane may fail to exist if $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ are not continuous. In other words, be careful when a denominator takes on a zero, or when function can’t make up its mind about a certain value.

Ex. Let $f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$. Find $f_x$ and $f_y$. Are they continuous?
The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$. Zoom in towards the $(0, 0, 0)$...
The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$. Zoom in towards the $(0, 0, 0)$, and,...
The Non-Tangent Plane

Let \( f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}} \). Zoom in towards the \((0, 0, 0)\), and nothing happens!
The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$ and zoom in towards $(0, 0, 0)$ on the graph of $\frac{\partial f}{\partial x}$. ...
The Non-Tangent Plane

Let \( f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}} \) and zoom in towards \((0, 0, 0)\) on the graph of \( \frac{\partial f}{\partial x} \) ...
The Non-Tangent Plane

Let \( f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}} \) and zoom in towards \((0, 0, 0)\) on the graph of \( \frac{\partial f}{\partial x} \) and EEEEEKKKK!!!!!
**Limits**

A function of two variables is called continuous at \((a, b)\) if

\[
\lim_{(x,y) \to (a,b)} f(x, y) = f(a, b).
\]

We say \(f\) is continuous in \(D\) if it is continuous at each point of \(D\).

**Example:** Show \(f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}\) is continuous at zero, but that \(f_x\) is not.