LECTURE OUTLINE

Power Series

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Math 8

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Goals

Power Series
Radius of Convergence
Differentiation and Integration of Power Series
**Power Series**

A function given by

\[ f(x) = \sum_{n=0}^{\infty} c_n(x - a)^n. \]

is called a power series. It domain is the set of \( x \) where this series converges.

Ex: Where does \( \sum_{n=1}^{\infty} x^n \) converge, and simplify this function where it converges.
Radius of Convergence

\[ f(x) = \sum_{n=0}^{\infty} c_n (x - a)^n \]
either
(a) converges only at \( a \)
(b) converges for all \( x \)
(c) there is an \( R > 0 \) called the radius of convergence such that
\( f(x) \) converges or all \( x \) such that \( a - R < x < a + R \) and
diverges for all \( a + R < x \) and \( x < a - R \).

Ex: Find the radius of convergence of

\[ \sum_{n=1}^{\infty} 2^n (x - 7)^n. \]
Differentiation and Integration

If \( f(x) = \sum_{n=0}^{\infty} c_n (x - a)^n \) has a radius of convergence \( R \), then

\[
\frac{df}{dx} = \sum_{n=1}^{\infty} n c_n (x - a)^{n-1}, \quad \text{and}
\]

\[
\int f \, dx = \sum_{n=0}^{\infty} \frac{c_n}{n+1} (x - a)^{n+1} + C
\]

in \((a - R, a + R)\), and each of these power series has radius of convergence \( R \).
Examples

Ex: Find a power series expansion of $\log(1 - x)$ about $x = 0$ and its radius of convergence.

Ex: Find a power series expansion of $\arctan(x)$ about $x = 0$ and find its radius of convergence.
Exciting Examples

Find the radius of convergence of the power series expansion of

\[
\frac{1}{1 + x}
\]

about \(a = 0, \ a = 1, \ \text{and} \ a = C\).

Explore the same question for

\[
\frac{1}{1 + x^2}.
\]