LECTURE OUTLINE
Integration by Parts

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Math 8

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Goal

Chain Rule, eluR niahC
Product Rule, eluR tcudorP
Trig Inverses
The chain rule assures us that

\[
\frac{d(f(u))}{dx} = \frac{df}{du}(u) \frac{du}{dx},
\]

hence we find the elur niahc

\[
\int \frac{df}{du}(u) \frac{du}{dx} \, dx = f(u(x)) + C.
\]
For pronunciation purposes, we express the elur niahc as

\[ \int h(u) \frac{du}{dx} dx = \int h(u) du \]

where \( f(u) = \int h(u) du \), and call its use \( u \)-substitution.
Example 1

Use \( u \)-substitution to find

\[
\int \frac{x}{1 + x^2} \, dx.
\]
Reversing the Product Rule

The product rule assures us that

$$(uv)' = u'v + uv',$$

hence we find elur tcudorp

$$\int uv' \, dx + \int u'v \, dx = uv + C.$$
For pronunciation purposes, we write the elur tcudorp as

$$\int uv'\,dx = uv - \int u'v\,dx$$

and call its use *integration by parts*.
Example 2

Use integration by parts to find

$$\int \ln(x) \, dx.$$
Let $\tan^{-1}(x)$ be an continuous inverse of $\tan(x)$, when $\tan(x)$ has been restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. 
Derivative of $\tan^{-1}$

Prove that

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1 + x^2},$$

and hence that

$$\int \frac{1}{1 + x^2} dx = \tan^{-1}(x) + C.$$
Example 3

Find

\[ \int \tan^{-1}(x) \, dx \]
Similarly for

\[
\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}
\]

\[
\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2} \quad \frac{d}{dx} \cot^{-1}(x) = \frac{-1}{1+x^2}
\]

\[
\frac{d}{dx} \sec^{-1}(x) = \frac{1}{x\sqrt{1-x^2}} \quad \frac{d}{dx} \csc^{-1}(x) = \frac{-1}{x\sqrt{1-x^2}}
\]
Example 4

Find the area of the region bounded by \( y = \frac{\pi^2}{4} e^{(-x+\pi)} \sin(x) \) and \( y = x^2 e^x \) and the lines \( x = 0 \), \( x = \frac{\pi}{2} \).