1. [20 points] Determine whether the following limit exists and, if it exists, find its value:

$$\lim_{(x,y) \to (1,1)} \frac{y \sin(x - 1)}{x + y - 2}.$$ 

2. [15 points] Superman flies through space with coordinate functions $x(t) = 2t^2, y(t) = -t^2, z(t) = -2t^2$. What is the total distance he travels from time $t = 0$ to $t = 2$?

3. [15 points] Find the general solution to the second-order differential equation

$$y'' - 5y' + 6y = 0.$$ 

4. [20 points] Let $f(x, y) = x^2 + xy + y^2$.

(a) Find the maximum and minimum values of $f$ along the circle $x^2 + y^2 = 1$.

(b) Find the extreme points and values for $f$ in the disk $D$ consisting of the points $(x, y)$ such that $x^2 + y^2 \leq 1$.

5. [20 points] Find the Taylor series of $e^{x^2}$ expanded around $a = 2$ and find its radius of convergence.

6. [20 points] Let $f(x, y, z) = x^2 + y^2 + z^2 - 2x - 2y - 2z + 3$, and let $S$ be a level surface of $f$ given by $f(x, y, z) = 4$. Let $P$ be a point $(1, 1, 3)$ on $S$.

(a) Find the equation of the plane that is tangent to $S$ at the point $P$.

(b) Find the parametric equations of the line that is normal to $S$ at $P$.

Justify your answers.

7. [20 points] Find all the local minima, maxima, and saddle points of

$$f(x, y) = x^3 + y^3 - x - 3y + 3.$$ 

Find also the values of $f$ at these points. Justify your answers.

8. [20 points] Let $f(x, y)$ be a differentiable function of two variables. Given a unit vector $\vec{u}$, let $D_{\vec{u}}f$ be the directional derivative of $f$ in the direction $\vec{u}$.

(a) For which direction $\vec{u}$ is $D_{\vec{u}}f$ maximal?

(b) What is the maximum value?

To get any credit for this problem, you must prove your answers.