1. (15) (Short Answer) Compute the following, if they exist. If not, explain why.

(i) Find the area of the parallelogram whose vertices are \((0, 0, 0), (1, 3, 1), (3, 1, 1),\) and \((4, 4, 2)\).

**Answer:** 
Area = \(|\langle 1, 3, 1 \rangle \times \langle 3, 1, 1 \rangle| = |\langle 2, 2, -8 \rangle| = \sqrt{4 + 4 + 64} = \sqrt{72} \)

(ii) Let \(\vec{r}(t)\) be the vector-valued function \(\vec{r}(t) = \langle t^2, e^t, \cos(t) \rangle\).

Find \(\lim_{h \to 0} \frac{\vec{r}(h) - \vec{r}(0)}{h} = \quad \).

**Answer:** 
\(\vec{r}'(t) = \langle 2t, e^t, -\sin(t) \rangle\).
\(\lim_{h \to 0} \frac{\vec{r}(h) - \vec{r}(0)}{h} = \vec{r}'(0) = \langle 0, 1, 0 \rangle\).

(iii) Find the distance between the point \((4, -3, 2)\) and the plane \(x + y + z = 0\).

**Answer:** 
\[D = \frac{|4 - 3 + 2|}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{3}{\sqrt{3}} = \sqrt{3} \]

(iv) The arc length \(L\) of the curve defined parametrically by
\[\langle x, y, z \rangle = \langle 13 \cos(t), 5 \sin(t), 12 \sin(t) \rangle, \quad 0 \leq t \leq 2\pi,\]
is given by \(L = \quad\).

**Answer:** 
\(\vec{r}'(t) = \langle -13 \sin(t), 5 \cos(t), 12 \cos(t) \rangle, \quad |\vec{r}'(t)| = \sqrt{169} = 13\).

\[L = \int_0^{2\pi} |\vec{r}'(t)| \, dt = \int_0^{2\pi} 13 \, dt = 26\pi \]

(v) If \(z = \ln(\sqrt{x^2 + y^2})\) then \(\frac{\partial z}{\partial x} = \quad\).

**Answer:** 
\(z = \ln(x^2 + y^2)^{\frac{1}{2}} = \frac{1}{2} \ln(x^2 + y^2)\).

\(\frac{\partial z}{\partial x} = \frac{1}{2} \frac{2x}{x^2 + y^2} = \frac{x}{x^2 + y^2} \)
2. (10) (True or False) Circle the correct answer for each of the following statements. Let \( \vec{u}, \vec{v} \) and \( \vec{w} \) denote arbitrary non-zero vectors.

False  (i) The cross product of two unit vectors is also a unit vector.
True   (ii) Two distinct planes in \( \mathbb{R}^3 \) that do not intersect in a line must be parallel.
False  (iii) If line \( L_1 \) is skew to line \( L_2 \) and \( L_2 \) is skew to line \( L_3 \) then \( L_1 \) must be skew to \( L_3 \).
True   (iv) The vectors \( \vec{v} \times \vec{u} \) and \( \vec{u} \times 3\vec{v} \) are parallel.
True   (v) If \( \vec{u} \cdot \vec{v} = -1 \) and \( \vec{u} \cdot \vec{w} = 3 \) then \( \vec{u} \) is perpendicular to the vector \( 6\vec{v} + 2\vec{w} \).
False  (vi) The normal vector to the plane \( z = 3x - 7y + 2 \) is the vector \( \vec{n} = \langle 3, -7, 2 \rangle \).
True   (vii) The direction vector of the line

\[
\frac{3-x}{2} = \frac{y-5}{1} = \frac{z-2}{3}
\]

is the vector \( \vec{v} = -2\vec{i} + \vec{j} + 3\vec{k} \).
True   (viii) The planes \( x - y + z = 0 \) and \( x + 2y + 3z = 6 \) intersect.
True   (ix) If a force \( \vec{F} \) is applied to a vector \( \vec{v} \) then the magnitude of the torque is \( |\vec{F}||\vec{v}||\sin(\theta) \), where \( \theta \) is the angle between \( \vec{v} \) and \( \vec{F} \).
False  (x) The mixed partial derivatives \( f_{xy} \) and \( f_{yx} \), if they exist, of a function \( f(x, y) \) are equal: \( f_{xy} = f_{yx} \).
3. (20) (Multiple Choice) Circle the correct answer for each of the following. No work need be shown for credit.

(i) The plane passing through the points \((0, 0, 0), (2, -4, 6),\) and \((5, 1, 3)\) is given by

A. \(\langle x, y, z \rangle \times \langle 3, 5, -3 \rangle = 0\)  
B. \(\langle x, y, z \rangle \cdot \langle 3, 5, -3 \rangle = 0\)  
C. \(\langle x, y, z \rangle \cdot [(5, 1, 3) \times (2, -4, 6)] = 0\)  
D. \(\langle x, y, z \rangle \times [(5, 1, 3) \cdot (2, -4, 6)] = 0\)

(ii) Which one of the following expressions does NOT describe a line passing through the origin?

A. \(\langle x, y, z \rangle = \langle 2t, t, -6t \rangle\)  
B. \(x + y + z = 0\)  
C. \(\frac{x-1}{2} = \frac{y-2}{4} = \frac{z+1}{2}\)  
D. \(x = 5t, y = \frac{3}{2}t, z = -\frac{3}{2}t\)

(iii) A drug manufacturer needs to purchase 4 kg of aluminum phosphate, 2 kg of benzalkonium chloride, and 1 kg of cromolyn sodium. These chemicals cost $250/kg, $86/kg, and $3061/kg respectively. Let \(\vec{v} = \langle 4, 2, 1 \rangle\) and \(\vec{w} = \langle 250, 86, 3061 \rangle\). Which of the following is the total cost of this purchase?

A. The cross product of \(\vec{v}\) and \(\vec{w}\).  
B. The triple product of \(\vec{v}\) and \(\vec{w}\).  
C. The dot product of \(\vec{v}\) and \(\vec{w}\).  
D. The cross product of \(\vec{w}\) and \(\vec{v}\)

(iv) The level curves (at height \(k\) above the xy-plane) for the function \(z = \sqrt{x^2 + y^2}\) are

A. lines \(y = (1 + k)x\) where \(0 \leq k < \infty\).  
B. circles of radius \(k\) centered at the origin.  
C. parabolas \(y = kx^2\) through the origin.  
D. circles of radius \(k^2\) centered at the origin.

(v) The domain of the function

\[f(x, y) = \sqrt{x^2 + y^2 - 1 + \ln(4 - x^2 - y^2)}\]

is equal to the set of all points \((x, y)\) in the plane whose distance \(r\) to the origin satisfies:

A. \(1 \leq r < 2\)  
B. \(1 < r < 4\)  
C. \(1 < r < 2\)  
D. \(1 < r \leq 4\)
4. (20) Consider the following two planes in space:

\[ P_1 : x + y - z = 0 \]
\[ P_2 : x - 3y + z = 0 \]

Please show your work and simplify answers, where possible, in order to receive full credit.

(i) What are the normal vectors \( \vec{n}_1 \) and \( \vec{n}_2 \) of planes \( P_1 \) and \( P_2 \), respectively?

**Answer:** \( \vec{n}_1 = \langle 1, 1, -1 \rangle \) and \( \vec{n}_2 = \langle 1, -3, 1 \rangle \).

(ii) Find the cosine of the angle between the two planes.

**Answer:** \( \cos(\theta) = \sqrt{\frac{3}{11}} \).

(iii) Find the cross product \( \vec{n}_1 \times \vec{n}_2 \).

**Answer:** \( \vec{n}_1 \times \vec{n}_2 = \langle -2, -2, -4 \rangle \).

(iv) Find a point \( P(a, b, c) \) on the line of intersection of these two planes whose \( z \) coordinate is \( c = 4 \).

**Answer:** \( P(2, 2, 4) \).

(v) Find the parametric equations of the line of intersection of planes \( P_1 \) and \( P_2 \).

**Answer:** \( x = 2 - 2t, y = 2 - 2t, z = 4 - 4t \).
5. (15) An electron is spiralling in a magnetic field. It’s velocity vector at time $t$ is given by

$$\vec{v}(t) = \langle 2t, -3\sin(t), -3\cos(t) \rangle$$

where time $t$ is in seconds and distance is in meters. It begins moving at time $t = 0$ from the point $P(-2, 3, 0)$. Please show your work and simplify answers, where possible, in order to receive full credit.

(i) At what time is the speed of the electron 5 meters/second?

**Answer:** Solve $|\vec{v}(t)| = \sqrt{4t^2 + 9} = 5 \implies t = 2$ seconds.

(ii) What is the acceleration vector $\vec{a}(t)$ of the electron?

**Answer:** $\vec{a}(t) = \langle 2, -3\cos(t), 3\sin(t) \rangle$

(iii) Find the position vector $\vec{r}(t)$ of the electron.

**Answer:** $\vec{r}(t) = \langle t^2 - 2, 3\cos(t), -3\sin(t) \rangle$

(iv) The magnetic field vector at time $t$ is given by

$$\vec{F}(t) = \langle t - 2, 2\sin(t), 2\cos(t) \rangle.$$ 

At what time(s) $t$ is the electron moving perpendicular to the magnetic field vector?

**Answer:** Solve $\vec{v} \cdot \vec{F} = 2t^2 - 4t - 6 = 2(t - 3)(t + 1) = 0$.

The electron is moving perpendicular to the magnetic field vector at time $t = 3$ seconds.
6. (20) Compute the following limits (if they exist) concerning the function

\[ f(x, y) = \frac{xy^3}{x^2 + 4y^6}. \]

Where appropriate, write DNE if the limit “does not exist” (and explain why) or NEIG if “not enough information is given” to compute the limit.

(i) The limit of \( f(x, y) \) as \((x, y)\) approaches the origin along a line of slope \( m \).

**Answer:** Along the line \( y = mx \) we have \( f(x, mx) = \frac{m^3x^4}{x^2 + 4m^6x^6} \).

\[
\lim_{(x,y)\to(0,0)} f(x, mx) = \lim_{x\to0} \frac{m^3x^4}{x^2 + 4m^6x^6} = 0
\]

(ii) The limit of \( f(x, y) \) as \((x, y)\) approaches the origin along the curve \( y = k\sqrt{x} \), where \( k > 0 \).

**Answer:** Along the curve \( y = k\sqrt{x} \) we have \( f(x, k\sqrt{x}) = \frac{k^3x^{5/2}}{x^2 + 4k^6x^3} \).

\[
\lim_{(x,y)\to(0,0)} f(x, k\sqrt{x}) = \lim_{x\to0} \frac{k^3x^{5/2}}{x^2 + 4k^6x^3} = 0
\]

(iii) The limit of \( f(x, y) \) as \((x, y)\) approaches the origin along the curve \( y = C\sqrt[3]{x} \), where \( C \neq 0 \).

**Answer:** Along the curve \( y = C\sqrt[3]{x} \) we have \( f(x, C\sqrt[3]{x}) = \frac{C^3}{1 + 4C^6} \).

\[
\lim_{(x,y)\to(0,0)} f(x, C\sqrt[3]{x}) = \lim_{x\to0} \frac{C^3}{1 + 4C^6} = \frac{C^3}{1 + 4C^6} \neq 0
\]

(iv) What is \( \lim_{(x,y)\to(0,0)} f(x, y) \)?

**Answer:** DNE. The limit does not exist because the limit in part (iii) differs from the limits in (i) and (ii).

(v) What is \( \lim_{(x,y)\to(1,-1)} f(x, y) \)?

**Answer:** Since \( f(x, y) \) is continuous at \((1, -1)\) we have

\[
\lim_{(x,y)\to(1,-1)} f(x, y) = f(1, -1) = -\frac{1}{5}
\]