1. (Optimization).
   (a) Find and classify all local extreme points of \( f(x, y) = x^2 + x + 2y^2 \) on the domain \( x^2 + y^2 < 1 \).
   (b) Determine the absolute maximum and minimum of \( f(x, y) = x^2 + x + 2y^2 \) on the domain \( x^2 + y^2 \leq 1 \). Be sure to indicate both the maximum and minimum values as well as the coordinates of all points at which they occur.

2. Suppose that \( z = f(x, y) \), \( x = uv \) and \( y = u + 3v \). Assume that when \( u = 2 \) and \( v = 1 \), \( \frac{\partial z}{\partial u} = -2 \) and \( \frac{\partial z}{\partial v} = -1 \). Find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \).

3. Find an equation of the plane which is perpendicular to the line \( x = 2 - t, y = 2t, z = 3 + t/2 \), and which contains the line \( x = 4 + 2s, y = -1 + 3s, z = 2 - 8s \).

4. Consider the surface \( x^2 + y^2 + z^2 = 9 \). Find the point of intersection of the tangent plane to the surface at the point \( (1, 2, 2) \) and the \( x \)-axis.

5. Find the maxima and minima of \( f(x, y, z) = xyz \) subject to the constraint \( g(x, y, z) = x^2 + 2y^2 + 3z^2 = 6 \).

6. Write an equation for the tangent plane to the level surface \( f(x, y, z) = ze^{xy} + xe^{yz} = 2 \) at the point \( (1, 0, 1) \).

7. What is the arclength of the curve \( y = \ln(\cos(x)) \) for \( x \) from 0 to \( \pi/4 \).

8. Find the absolute extrema of \( f(x, y) = e^{xy} + e^x \) in the first quadrant of the \( xy \)-plane.

9. Express the antiderivative \( \int \frac{\sin(t^2) - t^2}{t^6} \, dt \) as an infinite series.

10. (Multiple choice — No partial credit) **Circle the correct answer.**

   (a) Find the tangent plane to the surface \( z = x^2 y^3 \) at the point \( (1, 1, 1) \).
   - A. \( 2x + 3y - z = 4 \)
   - B. \( 3x + y - z = 3 \)
   - C. \( x + 2y + z = 4 \)
   - D. \( 2x + 3y = 5 \)
   - E. \( 3x - 2y + z = 2 \)

   (b) Consider the level curve of \( f(x, y) = x^2 - 3y^2 \) which passes through the point \( (3, 1) \). Along what vector should one go to remain on the same level curve?
   - A. \( (6, -6) \)
   - B. \( (-6, 6) \)
   - C. \( (-6, -6) \)
   - D. \( (0, 6) \)
   - E. \( (6, 0) \)
(c) What is the arclength of the piece of the parabola \( y = x^2 \) from \((0, 0)\) to \((2, 4)\)?

A. \( \int_{0}^{4} (1 + 2t) \, dt \)  
B. \( \int_{0}^{4} \sqrt{1 + 4t^2} \, dt \)  
C. \( \int_{0}^{2} \sqrt{t^2 + t^4} \, dt \)  
D. \( \int_{0}^{2} \sqrt{1 + 4t^2} \, dt \)  
E. none of the above

(d) If \( f(x, y) = \int_{y}^{x} \cos(t^3) \, dt \), then \( \frac{\partial f}{\partial y} = 

A. \( \cos(y^3) \)  
B. \( 3y^2 \cos(y^3) \)  
C. \( \sin(y^3) \)  
D. \( -3y^2 \sin(y^3) \)  
E. none of the above

(e) Suppose that you are given a function \( f(x, y) \) and vectors \( \mathbf{u} = \left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle \) and \( \mathbf{v} = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle \). If \( (D_u f)(x_0, y_0) = 2 \) and \( (D_v f)(x_0, y_0) = -1 \), then \( \frac{\partial f}{\partial x}(x_0, y_0) = 

A. \( \frac{1}{2} \)  
B. \( 1 \)  
C. \( 2 \)  
D. \( \sqrt{3} \)  
E. none of the above

(f) Suppose that the graph of \( z = f(x, y) \) represents the surface of a mountain, and you are standing at a point \((x_0, y_0, z_0)\) on the surface. You are told that the gradient of \( f \) at \((x_0, y_0)\) is \( \nabla f(x_0, y_0) = (1, 3) \). If you move in the direction of the gradient, what is your initial angle of elevation?

A. \( \tan^{-1} 3 \)  
B. \( \cos^{-1} 3 \)  
C. \( \tan^{-1} \sqrt{10} \)  
D. \( \cos^{-1} \sqrt{10} \)  
E. none of the above