1. Calculate \( \int_0^{0.05} \frac{\sin(2x)}{x} \, dx \) with error < 10\(^{-5}\). (You may leave your answer as a sum of a number of terms.)

2. \( \text{a} \) Find the Taylor series for the function \( f(x) = e^x \) centered around \( a = \ln 2 \).

   \( \text{b} \) Use the Taylor remainder formula and the series you have obtained in [a] to calculate \( e^{0.1+\ln 2} \) with error less than 10\(^{-4}\). You might want to use the fact that \( e^{0.1} < 2 \). (You may leave your answer as a sum of a number of terms.)

3. The acceleration of a particle moving in space is \( \mathbf{a}(t) = \sin(t) \mathbf{i} + \cos(t) \mathbf{j} \). The initial velocity of the particle is \( \mathbf{v}(0) = -\mathbf{i} \) and the initial position is \( \mathbf{r}(0) = -\mathbf{j} + 3\mathbf{k} \).

   \( \text{a} \) Find the position of the particle as a function of time: \( \mathbf{r}(t) = ? \)

   \( \text{b} \) Find the distance the particle traveled from \( t_0 = 1 \) to \( t_1 = 5 \).

4. Consider the depicted parallelogram determined by the vectors \( \mathbf{A} \) and \( \mathbf{B} \), and its diagonals, which are lightly drawn.

   \( \text{a} \) Assign directions to the diagonals, and express the resulting two vectors in terms of \( \mathbf{A} \) and \( \mathbf{B} \).

   \( \text{b} \) If \( \mathbf{A} \) and \( \mathbf{B} \) have the same length, show that the diagonals are orthogonal.

5. Consider the four points \( A = (1, 1, 0), B = (4, 0, 0), C = (1, 2, 2) \) and \( D = (2, 6, 4) \).

   \( \text{a} \) Find the distance from the point \( D \) to the plane containing \( A, B \) and \( C \).

   \( \text{b} \) Find the volume of the parallelepiped determined by the vectors \( \mathbf{A}\mathbf{B}, \mathbf{A}\mathbf{C} \) and \( \mathbf{A}\mathbf{D} \).

6. Show that the following limit does not exist:

   \[ \lim_{(x,y) \to (1,1)} \frac{y \sin(x - 1)}{x + y - 2}. \]

7. The position of a particle moving subject to a force \( \mathbf{F} \) is given by the vector-valued function \( \mathbf{r}(t) \); the velocity vector is \( \mathbf{v}(t) = \mathbf{r}'(t) \). The momentum vector is defined by \( \mathbf{p} = m\mathbf{v} \), where \( m \) is the particle’s mass; by Newton’s Law, \( \mathbf{F} = m\mathbf{v}' = \mathbf{p}' \). The angular momentum vector \( \mathbf{L} \) is defined by \( \mathbf{L} = \mathbf{r} \times \mathbf{p} \). Suppose that the force vector is always in the direction of the position vector \( \mathbf{r} \). Show that in this case angular momentum is conserved, i.e., that the angular momentum vector is constant. [Hint: Show that the derivative of \( \mathbf{L} \) is zero.]

8. How should the constant \( c \) be chosen so that the line \( \frac{x - 1}{2} = \frac{y + 1}{c} = \frac{z - 5}{3} \) is contained in the plane \( 3x - 2y = 5 \)?