Instructions:

- Answer ALL questions from Section A
- You may use a handwritten sheet of notes. Calculators are NOT permitted.
- Read all questions carefully
- Unless explicitly told otherwise, you should explain all your answers fully.
- Do NOT separate the pages of your exam.

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Score: 1
Section A: Answer ALL questions.

Problem A1: [15 pts]

(a) Find an equation for the plane that passes through the points \(A(1, 0, 1), B(2, -1, 0)\) and \(C(1, 3, 2)\).

(b) Does the line \(\vec{r}(t) = (1, 2, 1) + t(1, -1, -1)\) intersect the plane \(x + 2y - z = -2\)? (Justify your answer.)

(c) What is the distance from the point \(P(1, 2, 1)\) to the plane \(x - y + 3z = -2\)?
Problem A2: [16 pts]

(a) Find \( \int \frac{1}{x} \sqrt{1 + 4x^2} \, dx \).

(b) Evaluate the arc-length of the curve \( \vec{r}(t) = (t, 2e^t) \) between the points \((0, 2)\) and \((1, 2e)\).
Problem A3: [16 pts] Consider the function \( f(x, y) = ye^{\sin x} \).

(a) Find the gradient \( \nabla f \).

(b) Find the tangent plane to the surface \( z = f(x, y) \) at the point \( (0, 1, 1) \).

(c) Use the function \( f(x, y) \) to approximate \( (0.9)e^{\sin 0.2} \) as a fraction.

(d) Find a tangent vector (at \( (\pi/2, 2) \)) to the contour (level set) of \( f(x, y) \) that passes through the point \( (\pi/2, 2) \).
Problem A4: [16 pts] Consider the function \( f(x, y) = (x - 1)(x^2 + y^2) - 8x \).

(a) Find and classify all the critical points of \( f \).

(b) Find the absolute max and min of \( f(x, y) \) on the region \( x^2 + y^2 \leq 9 \).
Problem A5: [15 pts]

(a) What is the radius of convergence of the power series
\[ \sum_{n=2}^{\infty} (-1)^n \frac{4^n}{n+1} (x - 2)^{2n}. \]

(b) Expand \( \frac{1}{(3 + x)^2} \) as a power series. What is its radius of convergence?
Problem A6: [10 pts] Does the improper integral

\[ \int_{0}^{1} \frac{\ln x}{\sqrt{x}} \, dx \]

converge or diverge? If it converges, what does it converge to?
Problem A7: [12 pts] The probability of the bird seeing a worm depends upon its position in space according to the formula

\[ P(x, y, z) = \frac{\cos^2(x + y)}{1 + z^2}. \]

(a) A bird’s flight path is given by the curve

\[ \mathbf{r}(t) = (t, t^2 \cos t, e^{-t}). \]

At time \( t = 0 \), is the bird’s chance of spotting a worm increasing or decreasing?

(b) If the bird starts at its location at \( t = 0 \), in which direction should it fly to make its chances of finding lunch increase most rapidly? (Your answer should be a unit vector.)