Q-factor essentials sheet

Q-factor is a measure of ‘quality’ of a particular resonance. Each normal mode, i.e., mode of oscillation, of a system has two important things: its resonant frequency (sometimes called $f_0$), and its Q-factor.

Q is a way of saying many things all at once, all of which follow from each other:

- high Q = ‘high quality’ = rings for many cycles (long decay time) after being hit
- very little damping = peak width of frequency response is narrow
- peak width of frequency response is tall.

Now in more detail... There are 2 simple things you can do to a system: hit it once (shown on left side) or excite (drive) it at a given frequency and see how much it responds (shown on right side) ... 

DECAY AFTER SINGLE HIT

Oscillation with exponential decay in time.

$\tau = \text{how long to die to } 1/e, \text{ about } 37\%$

$happens to be the definition$

$Q = \frac{\pi}{\tau}$

RESPONSE TO DRIVING AT GIVEN FREQ

Low frequency response called $C$: all we’re saying is the peak height is $Q$ times this.

The peak width is $Q$ times narrower than the position of the peak in frequency.

Eg tuning fork has many thousands of cycles during decay time -> high Q, whereas a piece of jello (jelly) only wobbles a few times before settling down -> low Q. Therefore if you want to excite the jello you don’t have to get the driving frequency very accurate; with tuning fork you do.

It is amazing that all these properties are controlled through a single quantity, $Q$. 