Numerical Invariants of Topological Spaces and Continuous Mappings

Martin Arkowitz
Dartmouth College

Thursday, October 31, 2002
102 Bradley Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract
Can one assign an integer \( n(X) \) to a topological space \( X \) with the following properties: (1) \( n(X) \) captures a geometric or topological property of \( X \) (2) \( n(X) \) is computable (3) \( n(X) \) is a topological invariant, i.e., if \( X \) is homeomorphic to \( Y \), then \( n(X) = n(Y) \)? In the first half of the talk we survey several classical ways of doing this. These are (1) dimension theory which deals with the dimension of a space \( X \) (2) the Euler characteristic of \( X \) (3) the category of \( X \) which is the minimum number of contractible spaces needed to construct \( X \). In each case we define the invariant, work out examples and present some illustrative theorems. In the second half of the talk we generalize to continuous mappings \( f : X \rightarrow Y \) of spaces. We discuss two distinct ways to assign an integer to \( f \): (1) the Lefschetz number of \( f \) which is a generalization of the Euler characteristic and is used to determine if \( f \) has a fixed point (2) the category of \( f \) which generalizes the category of a space.

This talk is meant to be elementary and expository.

This talk should be accessible to graduate students.