Three-dimensional topology is a rich subject where various branches of mathematics intersect. Many decades of work culminating in Perelman's proof of Thurston’s geometrisation conjecture have shown that 3-dimensional manifolds $W$ are essentially determined by their fundamental groups $G(W)$, and an important problem is to describe the properties of $W$ in terms of $G(W)$. For instance, geometrisation implies that a closed, connected $W$ admits a hyperbolic structure if and only if $G(W)$ is infinite, freely indecomposable, and contains no $\mathbb{Z} + \mathbb{Z}$ subgroups. In this talk I will describe recent work on the L-space conjecture, which posits a quite surprising connection between three different properties of 3-manifolds, one algebraic, one topological, and one analytic.