Abstract

The isoperimetric profile of a Riemannian manifold is a function that measures the smallest amount of area needed to bound a specified volume within the manifold. Although this profile has many nice theoretical properties, it is difficult to compute in practice even for simple manifolds. Motivated by an application to political redistricting, we define a total variation relaxed version of this profile as an example of a multiscale compactness measure. We show that the new profile is the lower convex envelope of the isoperimetric profile and satisfies an isoperimetric inequality, as well as being efficiently computable. Additionally, we completely characterize the behavior of the TV-profile for small $t$ in terms of a Cheeger set for the manifold. I will also discuss various extension of this profile to arbitrary measures and graphs.

In addition to this compactness work, I will discuss several recent applications of mathematical methods for analyzing political redistricting, both in court cases and legislative reform efforts.