Infinite chess and the theory of infinite games

Joel David Hamkins
City University of New York

Thursday, January 23, 2014
008 Kemeny, 4:00PM
(3:30 Tea, 300 Kemeny)

Abstract

Using infinite chess as a central example—chess played on an infinite edgeless board—I shall give a general introduction to the theory of infinite games. Infinite chess is an example of an open game, a potentially infinite game which when won is won at a finite stage of play, and every open game admits the theory of transfinite ordinal game values, which measure the distance of a position to actual victory. When they are known, game values provide canonical winning strategies. I shall exhibit several interesting positions in infinite chess with high transfinite game values. The precise value of the omega one of chess, however, the supremum of all such ordinal game values, is not known; meanwhile, it is known in the case of infinite three-dimensional chess, where Evans and I proved that every countable ordinal arises as a game value. There are computable infinite positions in infinite chess that are winning for white, provided that black plays according to a computable procedure, but which is not winning for white when non-computable play is allowed. Also, the mate-in-n problem for finite positions in infinite chess is computably decidable (joint work with Schlicht, Brumleve and myself), despite the high quantifier complexity of any straightforward representation of it. See http://jdh.hamkins.org/infinite-chess-dartmouth-2014 for commentary and links.

This talk should be accessible to graduate students.