Spectral analysis of random-to-random Markov chains

Franco Saliola
Université du Québec à Montréal

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Abstract

"Pick a card (any card), remove it and put it back anywhere in the deck." Repeating this process defines a card shuffling technique known as the random-to-random shuffle. A natural question to ask of any shuffling technique is how many shuffles are needed to randomize the deck of cards. This is controlled by the spectra of the associated transition matrices.

By considering all the random-to-random shuffles simultaneously, we prove that the eigenspaces admit a beautiful recursive structure. This structure allows one to build eigenbases starting from bases for the kernels. Among other things, we obtain complete combinatorial descriptions of the eigenvalues of the transition matrices. The representation theory of the symmetric group features prominently in our analysis, but the results and the talk can be appreciated with no prior knowledge of representation theory.

This talk is based on joint work with Ton Dieker (Columbia University).

This talk should be accessible to graduate students.