Rigorous Results for Digital Snowflake Growth

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L02 Carson Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)
(In honor of J. Laurie Snell's 80^{th} Birthday)

Abstract

Cellular automata (CA) models for the evolution of snowflakes were introduced by Norman Packard in 1984, and have been popularized by Stephen Wolfram ever since. These are very simple two-dimensional algorithms with qualitative features of real snowflakes, but lacking many of the more subtle relevant effects of diffusive aggregation, temperature fluctuation, etc. Nevertheless, they are beautiful mathematical structures, exterior boundaries of which converge to the fractal known as the (von) Koch Snowflake that was introduced in 1904. For the past 20 years Wolfram has presented these automata as examples of complex dynamics generated by simple local interactions, with the suggestion that they are beyond the scope of rigorous mathematical methods.

In fact, the 16 Packard Snowflakes on the hexagonal lattice can be analyzed by traditional mathematical methods, albeit with invaluable help from computer visualization. For instance, we prove that each process has an asymptotic density as it fills the plane, independent of the finite initial seed, and we are able to compute the density exactly in 8 of the 16 cases. E.g., for the rule featured in the September 1984 *Scientific American*, Steven Levy's 1992 book *Artificial Life*, and Wolfram's postcards and various other publications, the density is 5/6. In the remaining 8 cases we give a convincing explanation why the density cannot be evaluated exactly, but show how (in principle) to derive bounds of arbitrary accuracy, and we determine the density to within .0001 for the most basic snowflake.

This talk should be accessible to graduate and advanced undergraduate students. 1105125706