Math 108. *Topics in combinatorics: The probabilistic method.*

Assignment 1. Due on Tuesday, 1/22/2008.

1. The bipartite Ramsey Number $BR(k)$ is the least n so that if A, B are disjoint with $|A| = |B| = m$ and $A \times B$ is 2-colored there exist $A_1 \subseteq A$, $B_1 \subseteq B$ with $|A_1| = |B_1| = k$ and $A_1 \times B_1$ monochromatic. Find and prove a theorem which gives a lower bound for $BR(k)$ and explore the asymptotics.

2. Find $m = m(n)$ as large as you can so that the following holds: Let $A_1, \ldots, A_m \subseteq \{1, \ldots, 4n\}$ with all $|A_i| = n$. Then there exists a 2-coloring of $\{1, \ldots, 4n\}$ such that none of the A_i is monochromatic. Express your answer as an asymptotic function of n.

3. Exercise 1, Page 10. (The first part was done in class. Just bound the Ramsey number $R(4, k)$.)

4. Exercise 2, Page 10.

5. Let $A = (a_{ij})$ be an $n \times n$ matrix with all $a_{ij} \in \{0, 1\}$. We call A *ninefree* if there is no 3×3 submatrix consisting of all ones. (Note: the rows and columns of a submatrix don't need to be consecutive.) Let $f(n)$ denote the maximal number of ones in a ninefree $n \times n$ matrix. Give a lower bound on $f(n)$ by the following argument: Consider a random matrix in which each coefficient is one with independent probability p. Now for each 3×3 submatrix consisting of all ones change one of the ones to a zero. This gives a ninefree matrix. Give a precise theorem giving a lower bound for $f(n)$ and then analyze the asymptotics.