MATH 116 WORKSHEET: Companion matrix

- SOLUTIONS -

A) Compute \( \det \left( \begin{array}{cc} z & a_2 \\ -1 & a_1z \end{array} \right) = z^2 + a_1z + a_2 \)

B) Compute \( \det \left( \begin{array}{ccc} z & a_3 \\ -1 & z \\ -1 & a_2 \end{array} \right) = z^3 + a_1z^2 + a_2z + a_3 \) (reusing the above).

C) A monic polynomial has leading coefficient 1. Show how to build a matrix whose eigenvalues are the roots of a general monic (i.e., non-monic) polynomial of degree \( p \):

[This is called Companion matrix] Since can divide coeffs by the leading coeff.

When \( p \) is odd, \( \det \left( \begin{array}{cccc} z & 1 & \cdots & a_{n-1} \\ -1 & z & \cdots & a_{n-2} \\ \vdots & \ddots & \ddots & \vdots \\ -1 & \cdots & -1 & z \end{array} \right) = 0 \) so \( A = \left( \begin{array}{cccc} 0 & 0 & \cdots & -a_n \\ 1 & 0 & \cdots & -a_{n-1} \\ 0 & 1 & \cdots & -a_{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{array} \right) \) has \( p \) as eigenvalue.

D) So, can there be a direct algorithm for the \( n \geq 5 \) general matrix EVD? No, since if there were, one could use it (as above) to give direct exact formula for roots of quintic & higher polynomials, which by Galois, Abel, etc. 1824, we know is impossible!
A) Compute \( \det \left( \begin{array}{cc} z & a_2 \\ -1 & a_1 + z \end{array} \right) \)

B) Compute \( \det \left( \begin{array}{cc} z & a_3 \\ -1 & z + a_2 \\ -1 & a_1 + z \end{array} \right) \)

reusing the above.

C) A monic polynomial has leading coefficient 1.

Show how to build a matrix whose eigenvalues are the roots of a general monic (i.e., monic) polynomial of degree \( p \).

[This is called a companion matrix.]

D) So, can there be a direct algorithm for the \( n \geq 5 \) general matrix EVP?