Math 126 Numerical PDEs: Homework 5—debriefing

February 16, 2012

1. [6 pts = 2 + 2 + 2]

   (a) There are various ways to make this work with a general R and Rp given function. One is make new inline funcs defined in terms of the old ones; see Lin, Brad. Jeff did vai text processing: harder to interpret, but the resulting funcs would evaluate faster since not so recursive.

   (b)

   (c) Well done with all your arrowheads - I didn't even know matlab had that built in! (get help on `quiver`)

2. [9 pts = 4+2+3]

   (a)

   (b) Important to communicate the size of error in the domain: show $\log_{10}|u^{(N)} - u|$ with a *labelled* color scale of $[-16, 0]$.

   (c) The rate is exponential, $e^{-\alpha N}$ with roughly $\alpha \approx 0.48$. Thus 75 pts needed for machine prec at the requested point. BONUS: the rate drops (worsens) as you approach the boundary. This could be fixed by adaptive quadrature or close-evaluation tricks; see suggested projects `projects.txt`

3. [6 pts = 4+2] I gave more points than quite necessary, since my estimate was more complicated that some of yours.

   (a) Here's my solution based on tricks of Lin and Brad (but who weren't quite correct). Only the first term depends on h, giving

   $$\left|\frac{d}{dh}\frac{\partial\Phi(x,y)}{\partial n_y}\right| = \left|\frac{\partial^2\Phi(x,y)}{\partial n_z \partial n_y}\right| \leq \frac{1}{2\pi|x-y|^2} \leq \frac{4}{2\pi|x-z|^2}$$

   So $C = 2/\pi$.

   (b) Now just integrating from zero to h, then using the crudest L^∞ bound in the surface integral, gives the desired with $C = 2|\partial\Omega|/\pi$.

4. [9 pts = 2+3+4]

   (a) This is hard since you have to code up curvature of your polar curve, which is a bit messy. See Brad's code `param.m` for a good example of doing this in polars. The best way to debug is to plot the kernel matrix vs i and j, that is, `imagesc(A)`, before the Id is added. Check it's smooth. Eg see Vipul's plot of this.

   (b) This is a matter of combining existing code pieces, now.

   (c) Error at requested point and node number should be -1.516×10^{-7}. Many of you got this!