- Prove JK2 from loc.11

**Other BVPs.**

We did Interior Dirichlet: "What temperature disk does uniformly conducting body settle to when body values set to some $f$?"

- Interior Neumann: $\Delta u = 0$ in $\Omega$, $\nabla u \cdot n = f$ on $\partial \Omega$. Equilibrium temp, dist. $f$ (specified heat input flux at each pt on $\partial \Omega$). What if pumping in more heat than extraction? Blow up!

Already knew a harm. = ZF: $\exists u_1, u_2, v \in \Omega$; $\Delta v = 0$ in $\Omega$, $v = f$ on $\partial \Omega$. Let $\omega = u_1 - u_2$ sat $\Delta \omega = 0$ in $\Omega$, $\omega = 0$ on $\partial \Omega$. Then if $u_1, u_2$ are solns, $w = u_1 - u_2$ sat $\Delta w = 0$ in $\Omega$, $w = 0$ on $\partial \Omega$. $w = 0$ is soln. (Only soln if $0 \in \text{spec}(\Delta_\Omega)$). 

The solution? if use $u = D \tau$ as before, BCs is $\int_\Omega \nabla \cdot (\nabla \tau \nabla \psi) \, dx = 0$.

Instead try? $u = S \tau$ so

$$\int_\Omega \nabla \cdot (\nabla \tau \nabla \psi) \, dx = 0 \quad \text{if} \quad \int_\Omega \nabla \cdot (\nabla \tau \nabla \psi) \, dx = 0$$

IE is $(I + 2D\tau)v = 2f$. 2nd kind again. $D \text{opt} \Rightarrow D \text{opt}$. But we know nonunique since BVP is, ask in practice, backwards stable lin solver should give $w = 0$; at least if $w = 0$ (p.36-7). A flaw here: $w = 0$ fails. $w = 0$ which satisfies. Danger: the const will be large as loss of digits in $\theta$.

Afternoon book: solve $(I + 2D\tau)$ $v = 2f$ when $x = 0$.

Proved uniquely solvable $\forall f C(\partial \Omega)$.

**Extend BVPs: eg. Dirichlet.**

$\Delta u = 0$ in $\mathbb{R}^2 \setminus \Omega$.

- $u = f$ on $\partial \Omega$.
- $\nabla u$ is bounded.

Has unique soln $\forall u C(\partial \Omega)$.

Follows from $\hat{u}(x) = \hat{u}(\frac{x}{|x|^2})$ "Khinchin rescale of $u$" maps $\mathbb{R}^2 \setminus \Omega$ to $\mathbb{R}$.

$\hat{u}$ is harmonic in $\Omega = \{ x : |x|^2 < \Delta x \}$ (only).

Condition at $x = 0$ is needed to control $\hat{u}$ at $0$.

More on for $s > 2$ have $o(1)$. Physically this indeed since potential at or needed to be specified. (Folland, PDE book.)