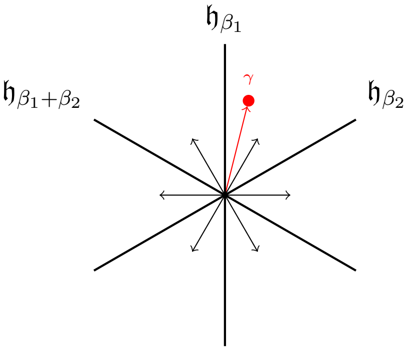
Math 128: Lecture 13

April 21, 2014

Existence of bases

A weight $\gamma \in \mathfrak{h}_{\mathbb{R}}^*$ is *regular* if $\gamma \notin \bigcup_{\alpha \in R} \mathfrak{h}_\alpha$.



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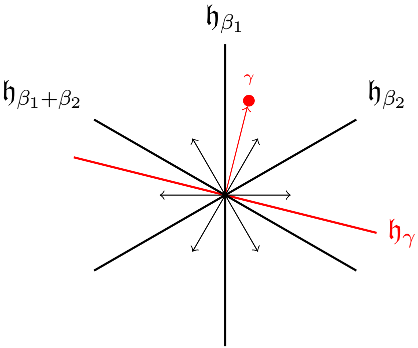

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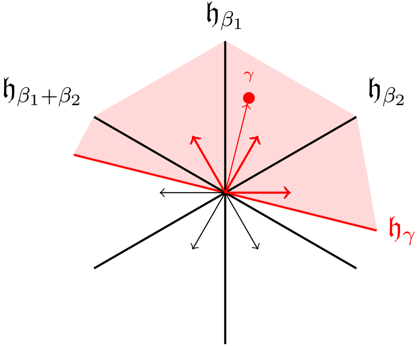

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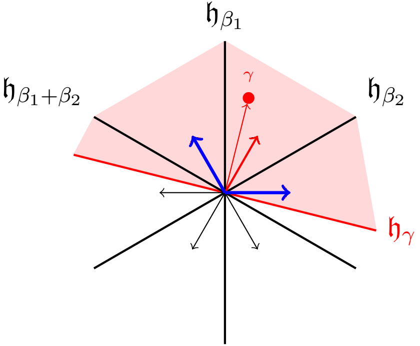

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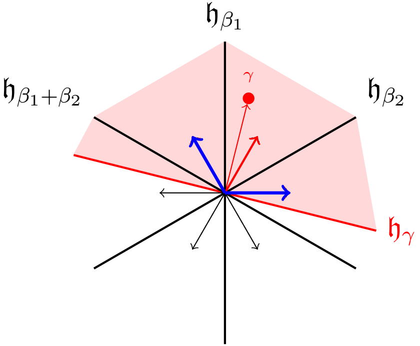
Let $B(\gamma) \subseteq R^+(\gamma)$ be the set of indecomposable roots in $R^+(\gamma)$.



Fix a fundamental chamber C, and therefore a base B and positive set of roots R^+. With $B = \{\beta_1, \ldots, \beta_r\}$, let $s_i = s_{\beta_i}$. Let W be the group generated by $\{s_\alpha \mid \alpha \in R\}$.

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Lemma

1. *The Weyl group W is finite.*
2. *The form $\langle , \rangle$ on $\mathfrak{h}_{\mathbb{R}}^*$ is W-invariant, i.e.*

$$\langle w(\alpha), \beta \rangle = \langle \alpha, w^{-1}(\beta) \rangle, \qquad \text{for all } \alpha, \beta \in R, w \in W.$$

3. *For all $\alpha \in R$, $w \in W$, we have $w s_\alpha w^{-1} = s_{w(\alpha)}$. Also, $w(\alpha^\vee) = w(\alpha)^\vee$.*
4. *The reflection associated to a simple root β setwise fixes $R^+ - \{\beta\}$ and $R^- - \{-\beta\}$.*
5. *If $w = s_{i_1} s_{i_2} \cdots s_{i_{\ell-1}}$ sends β_{i_ℓ} to a negative root, then $w s_{i_\ell} = s_{i_1} \cdots s_{i_{m-1}} s_{i_{m+1}} \cdots s_{i_{\ell-1}}$ for some $1 \le m < \ell$.*
6. *If $w = s_{i_1} s_{i_2} \cdots s_{i_\ell}$ with ℓ minimal, then $w(\beta_{i_\ell}) < 0$.*