

Dartmouth College  
Math. 12, Fall 1998

**PRACTICE FINAL EXAM**

1. Evaluate the integrals:

(a)  $\int x^2 \cos x dx$

(b)  $\int \frac{\cos x}{\sin^2 x} dx$

(c)  $\int \frac{x^2 + 2x}{(x + 1)^2} dx$

2. A certain population satisfies the growth law  $\frac{dP}{dt} = \frac{2t}{3}$ , where  $t$  is measured in years after January 1, 1990. On January 1, 1990, there were 500 people. How many will there be on January 1, 2000?

3. Determine the interval of convergence of the power series

(a)  $\sum_{n=1}^{\infty} \frac{5^n}{\sqrt{n}} x^n$

(b)  $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$

4. Find the sum of the series and state the interval on which this sum is valid

(a)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{n!}$

(b)  $\sum_{n=0}^{\infty} \frac{(x + 1)^{2n}}{9^n}$

5. Suppose that the series  $\sum_{n=1}^{\infty} a_n$  converges absolutely. Show that the series  $\sum_{n=1}^{\infty} \frac{n+1}{n} a_n$  also converges absolutely.

6. Use Maclaurin series to find  $f^{(10)}(0)$  for

(a)  $f(x) = x^7 e^{3x}$

(b)  $f(x) = \ln(1 - 3x)$

7. Points  $P(1, 1, 2)$ ,  $Q(1, 0, 1)$  and  $R(-1, -1, 0)$  determine a plane  $T$ .

(a) Find a unit vector which is orthogonal to the plane  $T$ .

(b) Find a vector equation of the line passing through  $(4, 0, -5)$  that is perpendicular to the plane  $T$ .

8. Use vectors to show that the line joining the midpoints of two sides of a triangle is parallel to the third side and is half as long.

9. Show that the lines  $\mathbf{r}_1 = \langle 2 - t, 4 + 2t, -3 + 4t \rangle$  and  $\mathbf{r}_2 = \langle 1 + t, 5 - 3t, 3 - 2t \rangle$  intersect, and find an equation of the plain that contains them.