Problem i) [Benjamin and Quinn, Section 7.7, Exercise 1] In class we proved that $H_n = \frac{c(n+1,2)}{n!}$ by counting certain permutations in two ways. Provide an alternative proof of this identity by following these steps:

- Write $H_n = \frac{a_n}{n!}$ (in other words, define $a_n = n!H_n$).
- Use the fact that $H_n = H_{n-1} + \frac{1}{n}$ to derive a recurrence for a_n.
- Prove that $a_n = c(n+1, 2)$ by showing that the both a_n and $c(n+1, 2)$ satisfy the same recurrence with the same initial conditions.

Problem j) [Benjamin and Quinn, Identity 196] Modify the proof seen in class of the identity

$$\sum_{k=1}^{n} k\, c(n,k) = c(n+1, 2)$$

to prove that

$$\sum_{k=m}^{n} c(n,k) \binom{k}{m} = c(n+1, m+1)$$

for any m and n.