Problem d) Prove that

$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$$

for $n \geq m \geq k \geq 0$ by counting something in two ways, and deduce that

$$\sum_{k=0}^{m} \binom{n}{k}\binom{n-k}{m-k} = 2^m \binom{n}{m}.$$