

Problem f) Let $c(n, k)$ denote the number $c(n, k)$ of permutations in S_n with k cycles. Find formulas for $c(n, n - 2)$ and for $c(n, 2)$, and double-check that they hold for $n = 4$.

Problem g) Let $S(n, k)$ the number of ways to partition the set $[n]$ into k disjoint non-empty subsets (that is, every element of $[n]$ has to be in exactly one of the subsets). For example, $S(4, 2) = 7$ because we can partition $[4]$ in the following ways:

$\{1, 2, 3\} \cup \{4\}, \{1, 2, 4\} \cup \{3\}, \{1, 3, 4\} \cup \{2\}, \{2, 3, 4\} \cup \{1\}, \{1, 2\} \cup \{3, 4\}, \{1, 3\} \cup \{2, 4\}, \{1, 4\} \cup \{2, 3\}.$

Find a recurrence for the numbers $S(n, k)$ similar to the one we gave for $c(n, k)$ in problem 56.