Midterm Exam 1
Math 1
October 4, 2012

Name: __________________________________________________________

Please circle your instructor’s name below:

Harnish Zhao

Please read the following instructions before starting the exam:

• This exam is closed-book, with no calculators, notes, or books allowed. You may not give or receive any help on the exam, though you may ask the instructors for clarification if necessary.

• Be sure to show all your work wherever possible. Even if your final answer is incorrect, we can assign an appropriate amount of partial credit if we can see how you arrived at your answer.

• Please circle or otherwise indicate your final answer.

• This test has a total of 10 questions, worth a total of 100 points. Point values are indicated for each question.

• You will have two hours from the start of the exam to complete it.

• Good luck!

Honor statement: I have neither given nor received any help on this exam, and I attest that all of the answers are my own work.

Signature: ______________________________________________________
This page is for grading purposes only.

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| Total   | 100    |       |
| Bonus   | 5      |       |
1. [12 pts] **Multiple choice.** Circle the correct answer for each question. Each part is worth 3 points.

(a) Which of the following functions is odd?
\[ f(x) = x^4 + x^3 \]
\[ g(x) = x^5 + x^{-1} + 1 \]

A. f(x)  
B. g(x)  
C. f(x) and g(x)  
D. neither

(b) Which of the following is the simplification of \( \ln 5 + 5 \ln 3 \)?

A. \( 5 \ln(5 \cdot 3) \)  
B. \( 5 \ln(5 + 3) \)  
C. \( \ln(5 \cdot 3^5) \)  
D. \( \ln(5 + 3^5) \)

(c) To get the graph of \( f(2x) \) from the graph of \( f(x) \), you would:

A. Stretch the graph of \( f \) vertically by a factor of 2.  
B. Stretch the graph of \( f \) horizontally by a factor of 2.  
C. Shrink the graph of \( f \) vertically by a factor of 2.  
D. Shrink the graph of \( f \) horizontally by a factor of 2.
(d) The following is the graph of \( f(x) \).

Which of the following is the graph of \( 2f(x + \frac{\pi}{4}) \)?
2. [5 pts] Simplify the following expression $\frac{b^3(2ab)^4}{a}$

3. [10 pts] $f(x) = e^x + 2$
   (a) [5 pts.] Find the inverse function of $f(x)$.

   (b) [5 pts.] Find the domain and range of $f^{-1}(x)$. 
4. [10 pts] \( f(x) = \begin{cases} 
  x & x \leq 0 \\
  x + 3 & x > 0
\end{cases} \)

(a) [4 pts.] Sketch the graph of \( f(x) \).

(b) [6 pts.] Find the domain and range of \( f(x) \).
5. [10 pts] A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side $x$ at each corner and then folding up the sides as in the figure. Express the volume $V$ of the box as a function of $x$. 


6. [10 pts] Simplify the following expressions:

(a) [2 pts] \( \log_2 16 \)

(b) [2 pts] \( \log_3 \frac{1}{27} \)

(c) [2 pts] \( \log_4 2 \)

(d) [4 pts] \( \log_3 6 - \log_3 8 + \log_3 12 \)
7. [6 pts] If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet $t$ seconds later is given by $y = 40t - 16t^2$.
Describe in words, DO NOT ACTUALLY CALCULATE, how you would go about finding the instantaneous velocity at $t = 2$. 
8. [18 pts] \( f(x) = \begin{cases} 
\cos x & x < 0 \\
3^x & 0 \leq x \leq 3 \\
\frac{1}{x-3} & x > 3 
\end{cases} \)

(a) [12 pts.] Draw the graph of \( f(x) \).

(b) [6 pts.] Evaluate the following limits. If the limit does not exist and is not infinite, explain why.

1. \( \lim_{x \to 3^+} f(x) \)

2. \( \lim_{x \to 3^-} f(x) \)

3. \( \lim_{x \to 3} f(x) \)

4. \( \lim_{x \to 0^+} f(x) \)

5. \( \lim_{x \to 0^-} f(x) \)

6. \( \lim_{x \to 0} f(x) \)
9. [15 pts] Evaluate the following limits.

(a) [5 pts] \( \lim_{x \to 2} \frac{x^2 - 7x + 9}{3 - x} \)

(b) [5 pts] \( \lim_{h \to 0} \frac{(-5 + h)^2 - 25}{h} \)

(c) [5 pts] \( \lim_{x \to -1} \frac{2x^2 + 3x + 1}{x^2 - 2x - 3} \)
10. [4 pts] If $2x \leq g(x) \leq x^4 - x^2 + 2$ for all $x$, evaluate $\lim_{x \to 1} f(x)$. You need to give the name or the statement the theorem you use to gain full credit.
**BONUS PROBLEM** [5 pts] Evaluate the following limit. If the limit does not exist and is not infinite, explain why.

\[ \lim_{x \to 0} \log_{1-x} (2 - x) \]