Homework 1 - Due September 19, 2012

1. A card is drawn from an ordinary deck of 52 cards. Let R be the event of drawing a red face card, let S be the event of drawing a spade, and let T be the event of drawing a ten.

    (a) Find $P(R)$.

    (b) Find $P(S \cup T)$.

    (c) Find $P(R \cup \widetilde{S})$.

2. (a) A fair 6-sided die is rolled twice. What is the probability that the first roll is higher than the second?

    (b) Repeat part (a) with an n-sided die. Find a formula in terms of n for the probability that the first roll is higher than the second. Explain your reasoning.

3. (Section 1.2, Problem 6) A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face. (For example, a six is three times as probable as a two.) What is the probability of getting an even number in one throw?

4. (Section 1.2, Problem 10) For a bill to come before the President of the United States, it must be passed by both the House of Representatives and the Senate. Assume that, of the bills presented to these two bodies, 60 percent pass the House, 80 percent pass the Senate, and 90 percent pass at least one of the two. Calculate the probability that the next bill presented to the two groups will come before the President.

5. (Section 1.2, Problem 20) Explain why it is not possible to define a uniform distribution function (see Definition 1.3) on a countably infinite sample space. Hint: Assume $m(\omega) = a$ for all ω, where $0 \leq a \leq 1$. Does $m(\omega)$ have all the properties of a distribution function?

6. (Section 3.1, Problem 6) In arranging people around a circular table, we take into account their seats relative to each other, not the actual position of any one person. Show that n people can be arranged around a circular table in $(n-1)!$ ways.

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7. (Section 3.1, Problem 12) A symphony orchestra has in its repertoire 30 Haydn symphonies, 15 modern works, and 9 Beethoven symphonies. Its program always consists of a Haydn symphony followed by a modern work, and then a Beethoven symphony.

   (a) How many different programs can it play?

   (b) How many different programs are there if the three pieces can be played in any order?

   (You do not need to complete part (c).)

Practice problems NOT to turn in: 1.2.2, 1.2.3, 1.2.8, 1.2.18(b), 1.2.31, 3.1.3, 3.1.7, 3.1.10. Remember that answers to odd-numbered questions are available online.